

## Chapter- 6

# TRIANGLES

**STUDY NOTES****SIMILAR FIGURES**

- Two figures having the same shape but not necessary the same size are called similar figures.
- All congruent figures are similar, but all similar figures are not congruent.

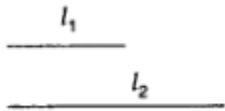
**SIMILAR POLYGONS**

Two polygons are said to be similar to each other, if:

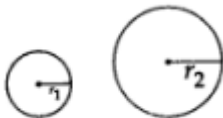
- (i) their corresponding angles are equal, and
- (ii) the lengths of their corresponding sides are proportional.

**Example:**

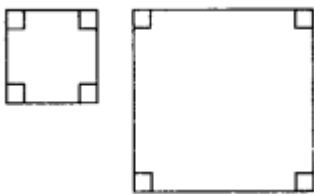
Any two line segments are similar since length are proportional



Any two circles are similar since radii are proportional



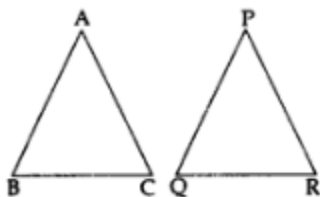
Any two squares are similar since corresponding angles are equal and lengths are proportional.

**Note:**

Similar figures are congruent if there is one to one correspondence between the figures.

∴ From above we deduce:

Any two triangles are similar, if their



(i) Corresponding angles are equal

$$\angle A = \angle P$$

$$\angle B = \angle Q$$

$$\angle C = \angle R$$

(ii) Corresponding sides are proportional

$$AB/PQ = AC/PR = BC/QR$$

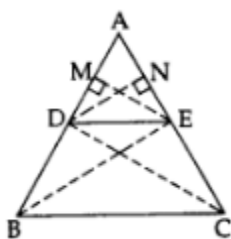
### THALES THEOREM OR BASIC PROPORTIONALITY THEORY

#### Theorem 1:

State and prove Thales' Theorem.

Statement:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given: In  $\triangle ABC$ ,  $DE \parallel BC$ .

To prove:  $AD/DB = AE/EC$

Const.: Draw  $EM \perp AD$  and  $DN \perp AE$ . Join B to E and C to D.

Proof: In  $\triangle ADE$  and  $\triangle BDE$ ,

$$\text{ar}(\triangle ADE)/\text{ar}(\triangle BDE) = \frac{1/2 \times AD \times EM}{1/2 \times DB \times EM} = AD/DB \dots\dots(i) \quad [\text{Area of } \Delta = 1/2 \times \text{base} \times \text{corresponding altitude}]$$

In  $\triangle ADE$  and  $\triangle CDE$ ,

$$\text{ar}(\triangle ADE)/\text{ar}(\triangle CDE) = \frac{1/2 \times AE \times DN}{1/2 \times EC \times DN} = AE/EC$$

$\therefore DE \parallel BC \dots$ [Given]

$\therefore \text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$

...[∵ As on the same base and between the same parallel sides are equal in area

From (i), (ii) and (iii),

$$AD/DB=AE/EC$$

### CRITERION FOR SIMILARITY OF TRIANGLES

Two triangles are similar if either of the following three criterion's are satisfied:

- **AAA similarity Criterion.** If two triangles are equiangular, then they are similar.
- **Corollary(AA similarity).** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- **SSS Similarity Criterion.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SAS Similarity Criterion.** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Results in Similar Triangles based on Similarity Criterion:

1. Ratio of corresponding sides = Ratio of corresponding perimeters
2. Ratio of corresponding sides = Ratio of corresponding medians
3. Ratio of corresponding sides = Ratio of corresponding altitudes
4. Ratio of corresponding sides = Ratio of corresponding angle bisector segments.

### AREA OF SIMILAR TRIANGLES

#### Theorem 2.

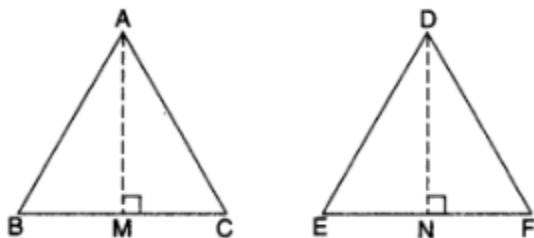
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given:  $\triangle ABC \sim \triangle DEF$

To prove:  $ar(\triangle ABC)/ar(\triangle DEF)=AB^2/DE^2=BC^2/EF^2=AC^2/DF^2$

Const.: Draw  $AM \perp BC$  and  $DN \perp EF$ .

Proof: In  $\triangle ABC$  and  $\triangle DEF$



$ar(\triangle ABC)/ar(\triangle DEF)=1/2 \times BC \times AM / 1/2 \times EF \times DN = BC/EF \cdot AM/DN$  ... (i) ..... [Area of  $\Delta = 1/2 \times \text{base} \times \text{corresponding altitude}$

$\therefore \triangle ABC \sim \triangle DEF$

$\therefore AB/DE = BC/EF$  .....(ii) ...[Sides are proportional]

$\angle B = \angle E$  .....[ $\therefore \triangle ABC \sim \triangle DEF$ ]

$\angle M = \angle N$  .....[each  $90^\circ$ ]

$\therefore \triangle ABM \sim \triangle DEN$  .....[AA similarity]

$\therefore AB/DE = AM/DN$  .....(iii) ...[Sides are proportional]

From (ii) and (iii), we have:  $BC/EF = AM/DN$  ... (iv)

From (i) and (iv), we have:  $\text{ar}(\triangle ABC)/\text{ar}(\triangle DEF) = BC/EF \cdot BC/EF = BC^2/EF^2$

Similarly, we can prove that

$\text{ar}(\triangle ABC)/\text{ar}(\triangle DEF) = AB^2/DE^2 = AC^2/DF^2$

$\therefore \text{ar}(\triangle ABC)/\text{ar}(\triangle DEF) = AB^2/DE^2 = BC^2/EF^2 = AC^2/DF^2$

### Results based on Area Theorem:

1. Ratio of areas of two similar triangles = Ratio of squares of corresponding altitudes
2. Ratio of areas of two similar triangles = Ratio of squares of corresponding medians
3. Ratio of areas of two similar triangles = Ratio of squares of corresponding angle bisector segments.

### Note:

If the areas of two similar triangles are equal, the triangles are congruent.

### Theorem 3:

**State and prove Pythagoras' Theorem.**

**Statement:**

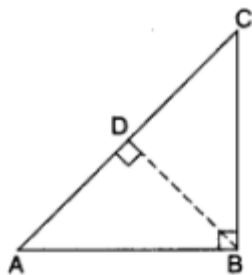
**Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.**

Given:  $\triangle ABC$  is a right triangle right-angled at B.

To prove:  $AB^2 + BC^2 = AC^2$

Const.: Draw  $BD \perp AC$

Proof: In  $\triangle s$  ABC and ADB,



$\angle A = \angle A$  ...[common

$\angle ABC = \angle ADB$  ...[each  $90^\circ$

$\therefore \triangle ABC \sim \triangle ADB$  ...[AA Similarity

$\therefore AB/AD = AC/AB$  .....[sides are proportional]

$\Rightarrow AB^2 = AC \cdot AD$

Now in  $\triangle ABC$  and  $\triangle BDC$

$\angle C = \angle C$  .....[common]

$\angle ABC = \angle BDC$  ....[each  $90^\circ$ ]

$\therefore \triangle ABC \sim \triangle BDC$  .....[AA similarity]

$\therefore BC/DC = AC/BC$  .....[sides are proportional]

$BC^2 = AC \cdot DC$  ... (ii)

On adding (i) and (ii), we get

$AB^2 + BC^2 = AC \cdot AD + AC \cdot DC$

$\Rightarrow AB^2 + BC^2 = AC \cdot (AD + DC)$

$AB^2 + BC^2 = AC \cdot AC$

$\therefore AB^2 + BC^2 = AC^2$

#### CONVERSE OF PYTHAGORAS THEOREM

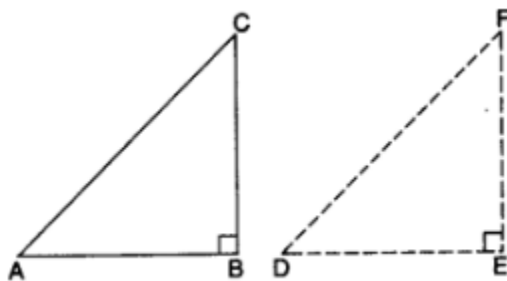
*Changing your tomorrow* 

##### Theorem 4:

State and prove the converse of Pythagoras' Theorem.

**Statement:**

Prove that, in a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given: In  $\triangle ABC$ ,  $AB^2 + BC^2 = AC^2$

To prove:  $\angle ABC = 90^\circ$

Const.: Draw a right angled  $\triangle DEF$  in which  $DE = AB$  and  $EF = BC$

Proof: In  $\triangle ABC$ ,

$$AB^2 + BC^2 = AC^2 \dots(i) \text{ [given]}$$

In rt.  $\triangle DEF$

$$DE^2 + EF^2 = DF^2 \dots[\text{by pythagoras theorem}]$$

$$AB^2 + BC^2 = DF^2 \dots(ii) \dots[DE = AB, EF = BC]$$

From (i) and (ii), we get

$$AC^2 = DF^2$$

$$\Rightarrow AC = DF$$

Now,  $DE = AB \dots[\text{by cont}]$

$EF = BC \dots[\text{by cont}]$

$DF = AC \dots\dots[\text{proved above}]$

$\therefore \triangle DEF \cong \triangle ABC \dots\dots[\text{sss congruence}]$

$\therefore \angle DEF = \angle ABC \dots\dots[\text{CPCT}]$

$\angle DEF = 90^\circ \dots[\text{by cont}]$

$\therefore \angle ABC = 90^\circ$

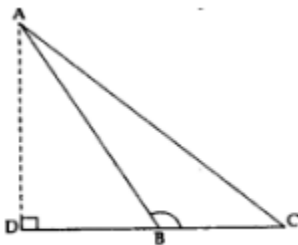
### Results based on Pythagoras' Theorem:

#### (i) Result on obtuse Triangles.

If  $\triangle ABC$  is an obtuse angled triangle, obtuse angled at B,

If  $AD \perp CB$ , then

$$AC^2 = AB^2 + BC^2 + 2 BC \cdot BD$$



#### (ii) Result on Acute Triangles.

If  $\triangle ABC$  is an acute angled triangle, acute angled at B, and  $AD \perp BC$ , then

$$AC^2 = AB^2 + BC^2 - 2 BD \cdot BC.$$

