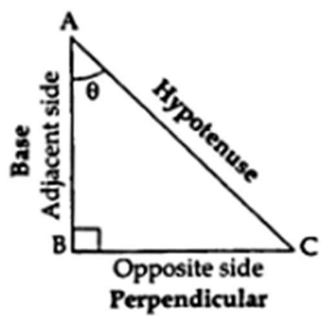
Chapter- 8
INTRODUCTION TO TRIGONOMETRY

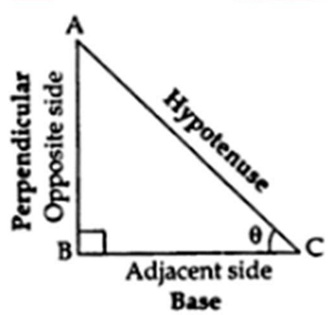
STUDY NOTES

- Trigonometry is the science of relationships between the sides and angles of a right-angled triangle.
- Trigonometric Ratios: Ratios of sides of right triangle are called trigonometric ratios.
  Consider triangle ABC right-angled at B. These ratios are always defined with respect to acute angle 'A' or angle 'C.
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of an angle can be easily determined.
- How to identify sides: Identify the angle with respect to which the t-ratios have to be calculated. Sides are always labelled with respect to the 'θ' being considered.

Let us look at both cases:



Case I: $\angle A = \theta$



Case II: $\angle C = \theta$

case I	case II
(i) sine A = perpendicular/hypotenuse=BC/AC	(i) sine C = perpendicular/Hypotenuse=ABAC
(ii) cosine A = base/hypotenuse=AB/AC	(ii) cosine C = base/hypotenuse=BC/AC
(iii) tangent A = perpendicular/base=BC/AB	(iii) tangent C = perpendicular/base=AB/BC

(iv) cosecant A = hypotenuse/perpendicular=AC/BC	(iv) cosecant C = hypotenuse/perpendicular=AC/AB
(v) secant A = hypotenuse/base=AC/AB	(v) secant C = hypotenuse/base=AC/BC
(v) cotangent A = base/perpendicular=AB/BC	(v) cotangent C = base/perpendicular=BC/AB

TRIGONOMETRIC IDENTITIES

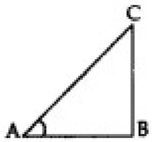
An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

$\tan \theta = \sin\theta\cos\theta$

$\cot \theta = \cos\theta\sin\theta$

- $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$
- $\text{cosec}^2 \theta - \cot^2 \theta = 1 \Rightarrow \text{cosec}^2 \theta = 1 + \cot^2 \theta \Rightarrow \cot^2 \theta = \text{cosec}^2 \theta - 1$
- $\sec^2 \theta - \tan^2 \theta = 1 \Rightarrow \sec^2 \theta = 1 + \tan^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1$
- $\sin \theta \,\text{cosec}\, \theta = 1 \Rightarrow \cos \theta \sec \theta = 1 \Rightarrow \tan \theta \cot \theta = 1$

A t-ratio only depends upon the angle ‘θ’ and stays the same for same angle of different sized right triangles.

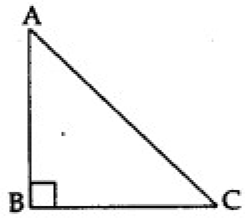


Value of t-ratios of specified angles:

∠A	**0°**	**30°**	**45°**	**60°**	**90°**
sin A	0	12	12√	3√2	1
cos A	1	3√2	12√	12	0
tan A	0	13√	1	√3	not defined
cosec A	not defined	2	√2	23√	1
sec A	1	23√	√2	2	not defined
cot A	not defined	√3	1	13√	0

The value of sin θ and cos θ can never exceed 1 (one) as opposite side is 1. Adjacent side can never be greater than hypotenuse since hypotenuse is the longest side in a right-angled Δ.

't-RATIOS' OF COMPLEMENTARY ANGLES



If $\triangle ABC$ is a right-angled triangle, right-angled at B, then
$\angle A + \angle C = 90°$ [$\because \angle A + \angle B + \angle C = 180°$ angle-sum-property]
or $\angle C = (90° - \angle A)$

Thus, $\angle A$ and $\angle C$ are known as complementary angles and are related by the following relationships:
$\sin (90° - A) = \cos A$; $\text{cosec} (90° - A) = \sec A$
$\cos (90° - A) = \sin A$; $\sec (90° - A) = \text{cosec} A$
$\tan (90° - A) = \cot A$; $\cot (90° - A) = \tan A$