

AREAS RELATED TO CIRCLES

PPT-4

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 12
CHAPTER NAME : AREAS RELATED TO CIRCLES

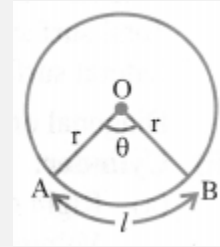
CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

Length of an Arc and Area of Sector

(i) The length of an arc of a sector of an angle θ is given by, $\frac{2\pi R\theta}{360^\circ}$

The area of the sector $A = \frac{\theta}{360^\circ} \times \pi r^2$ & θ is given by,



Area of Segment

i) Area of segment APB = Area (sector OAPB) – Area(Δ OAB)

This is the area of minor segment.

\therefore area of major segment AQB = πr^2 – Area of minor segment APB

(ii) If θ is the central angle, then the area of segment APB

$$= \frac{\theta}{360} \times \pi r^2 - r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

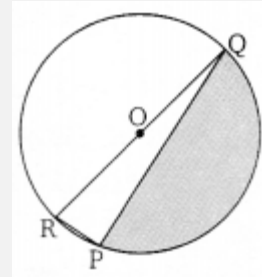
LEARNING OUTCOME

- 1 .Students will be able to find the area of combined plane figures.
- 2.Students will be able to identify angle subtended by the sector at the Centre.
3. Students will be able to apply the knowledge of Area of sector and segment of a circle in solving real life problems..

Area of combined plane figures

<https://youtu.be/KwfYxUPpJEY> {9.54}

1. Find the area of the shaded region in the given figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle)



1. Find the area of the shaded region in the given figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle)

In ΔPQR , $\angle QPR = 90^\circ$ (angle in semicircle)

and $QR^2 = PQ^2 + RP^2$

(as ΔQPR is a right angled triangle)

$$\therefore OR^2 = (24)^2 + (7)^2 = 576 + 49$$

$$\Rightarrow QR^2 = 625 \text{ cm}^2$$

$$\Rightarrow QR = \sqrt{625} \text{ cm}^2$$

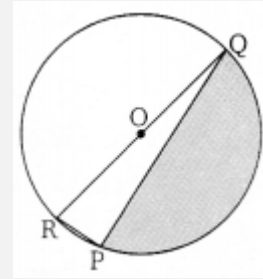
$$\Rightarrow QR = 25$$

Area of the shaded region = Area of the semicircle – Area of the triangle

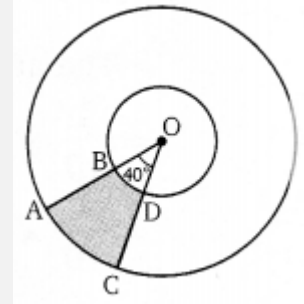
$$= \frac{1}{2} \pi r^2 - \text{area of } \Delta PQR = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times PR \times QP$$

$$= \frac{22}{7} \times \frac{25 \times 25}{2 \times 4} - \frac{1}{2} \times 7 \times 24 = \frac{22 \times 625}{28 \times 2} - 84 = \frac{13750 - 4704}{56}$$

$$= \frac{9046}{56} = \frac{4523}{28} = 161.54 \text{ cm}^2$$



2. Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$



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The Radius of smaller circle = 7cm

Angle of sector (θ) = 40°

\therefore Area of smaller sector BOD

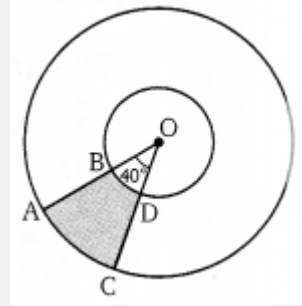
$$\begin{aligned} &= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7 \\ &= \frac{1}{9} \times 22 \times 7 = \frac{154}{9} \text{ cm}^2. \end{aligned}$$

\therefore Area of bigger sector AOC

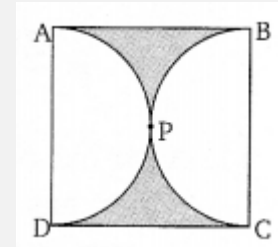
$$\begin{aligned} &= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \\ &= \frac{1}{9} \times 22 \times 28 = \frac{616}{9} \text{ cm}^2. \end{aligned}$$

\therefore Area of shaded region

$$\begin{aligned} &= \text{Area of bigger sector AOC} \\ &\quad - \text{Area of smaller sector BOD} \\ &= \frac{616}{9} - \frac{154}{9} = \frac{462}{9} = \frac{154}{3} \text{ cm}^2. \end{aligned}$$



3. Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



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ABCD is a square

Given: side of the square = 14 cm

Area of the square = (side)² = (14)² = 196 cm²

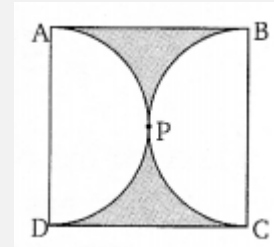
Radius of the semicircle APD = $\frac{1}{2}$ (side of square) = $\frac{1}{2} \times 14 = 7$ cm

$$\text{Area of the semicircle APD} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 11 \times 7 = 77 \text{ cm}^2$$

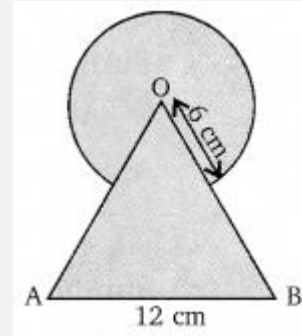
Similarly, area of the semicircle BPC = 77 cm²

Total area of both the semicircles = 77 + 77 = 154 cm²

Area of the shaded region = Area of square – area of both semicircles
= 196 – 154 = 42 cm²



4. Find the area of the shaded region in the figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as Centre.



4. Find the area of the shaded region in the figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as Centre.

Area of the equilateral triangle OAB

$$\begin{aligned}
 &= \frac{\sqrt{3}}{4} (\text{Side})^2 \\
 &= \frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3} \text{ cm}^2
 \end{aligned}$$

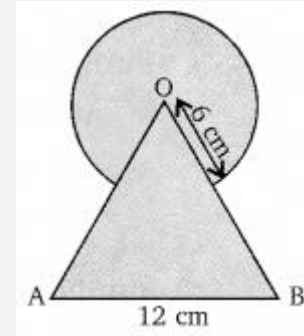
Angle of sector, $\theta = 60^\circ$ [Given]

Area of the major sector of the circle

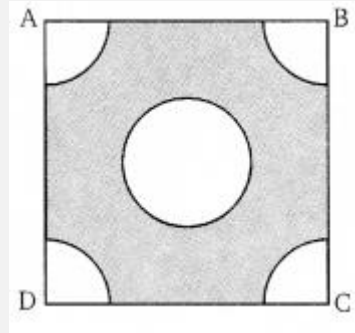
$$\begin{aligned}
 &= \pi r^2 - \text{Area of the minor sector} \\
 &= \pi \times 6 \times 6 - \pi \times 6 \times 6 \times \frac{60^\circ}{360^\circ} \\
 &= 36\pi - 6\pi = 30\pi = 30 \times \frac{22}{7} = \frac{660}{7} \text{ cm}^2
 \end{aligned}$$

Area of the shaded region

$$\begin{aligned}
 &= \text{Area of the major sector of the circle} \\
 &\quad + \text{Area of the triangle} \\
 &= \left(\frac{660}{7} + 36\sqrt{3} \right) \text{ cm}^2.
 \end{aligned}$$



5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the figure. Find the area of the remaining portion of the square.



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side of the square ABCD = 4 cm

Area of the square ABCD = $4 \times 4 = 16 \text{ cm}^2$

Radius of the quadrant at corner = 1 cm

$$\text{Area of the quadrant at each corner} = \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{1 \times 1 \times 90^\circ}{360^\circ} = \frac{22}{28} \text{ cm}^2$$

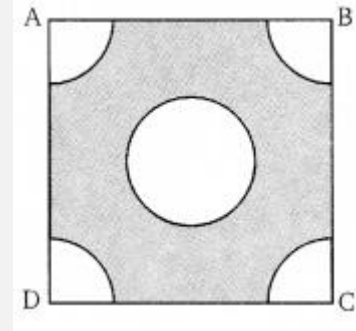
$$\text{Now, area of the 4 sectors at each corner} = \frac{4 \times 22}{28} = \frac{22}{7} \text{ cm}^2$$

Area of the circle at the centre of the square having (radius = $\frac{2}{2} = 1 \text{ cm}$)

$$= \pi r^2 = \frac{22}{7} (1)^2 = \frac{22}{7} \text{ cm}^2$$

$$\begin{aligned} \text{Total area to be cut out from square} &= \text{Area of the 4 sectors} \\ &\quad + \text{Area of the circle at the centre} \\ &= \frac{22}{7} + \frac{22}{7} = \frac{44}{7} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of the remaining portion} &= \text{Area of the square} - \text{Area to be cut from square} \\ &= 16 - \left(\frac{44}{7}\right) = 16 - \frac{44}{7} \\ &= \frac{112 - 44}{7} = \frac{68}{7} \text{ cm}^2 \end{aligned}$$



HOME ASSIGNMENT: Ex-12.3 Q1 to Q6 AHA

1. From each corner of a square of side 10 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut. Find the area of the remaining portion of the square.

THANKING YOU
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