

QUADRATIC EQUATIONS

PPT2

SUBJECT: MATHEMATICS

CHAPTER NUMBER: 04

CHAPTER NAME : QUADRATIC EQUATIONS

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

Quadratic Equation

When we equate a quadratic polynomial to a constant, we get a quadratic equation.

Any equation of the form $p(x) = ax^2 + bx + c$, where $p(x)$ is a polynomial of degree 2 and c is a constant, is a quadratic equation.

The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

' a ' is the coefficient of x^2 . It is called the quadratic coefficient. ' b ' is the coefficient of x . It is called the linear coefficient. ' c ' is the constant term.

LEARNING OUTCOME

1. Students will be able to find roots of a quadratic equation by factorization
2. Students will be able to represent situations in the form of Quadratic Equations.

Roots of a Quadratic equation

The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

If α is a root of the quadratic equation $ax^2 + bx + c = 0$, then $a\alpha^2 + b\alpha + c = 0$.

: [https://youtu.be/UtReXKgmQI0\(4.11\)](https://youtu.be/UtReXKgmQI0(4.11))

- **Solving a Quadratic Equation by Factorization method**

- Consider a quadratic equation $2x^2 - 5x + 3 = 0$

$$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$$

This step is splitting the middle term

We split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x^2 and the constant.

$$(-2) + (-3) = (-5)$$

$$\text{And } (-2) \times (-3) = 6$$

$$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$$

$$\Rightarrow 2x(x - 1) - 3(x - 1) = 0$$

$$\Rightarrow (x - 1)(2x - 3) = 0$$

In this step, we have expressed the quadratic polynomial as a product of its factors.

Thus, $x = 1$ and $x = 3/2$ are the roots of the given quadratic equation.

This method of solving a quadratic equation is called the factorization method.

: Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Solution : We have $6x^2 - x - 2$

$$= 6x^2 + 3x - 4x - 2$$

$$= 3x(2x + 1) - 2(2x + 1)$$

$$= (3x - 2)(2x + 1)$$

The roots of $6x^2 - x - 2 = 0$ are the values of x for which $(3x - 2)(2x + 1) = 0$

Therefore, $3x - 2 = 0$ or $2x + 1 = 0$, i.e., $x = 2/3$ or $x = -1/2$ Therefore, the roots of $6x^2 - x - 2 = 0$ are $2/3$ and $-1/2$

We verify the roots, by checking that $2/3$ and $-1/2$ satisfy $6x^2 - x - 2 = 0$

Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

$$\begin{aligned}\text{Solution : } & 3x^2 - 2\sqrt{6}x + 2 \\ &= 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 \\ &= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) \\ &= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2})\end{aligned}$$

So, the roots of the equation are the values of x for which

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\text{Now, } \sqrt{3}x - \sqrt{2} = 0 \text{ for } x = \sqrt{\frac{2}{3}}.$$

So, this root is repeated twice, one for each repeated factor $\sqrt{3}x - \sqrt{2}$.

Therefore, the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are $\sqrt{\frac{2}{3}}$, $\sqrt{\frac{2}{3}}$.

. Find the roots of the following quadratic equations by factorization

(ii) $2x^2 + x - 6 = 0$

(v) $100x^2 - 20x + 1 = 0$

(ii) We have,

$$2x^2 + x - 6 = 0 \quad \Rightarrow \quad 2x^2 + 4x - 3x - 6 = 0$$
$$\Rightarrow 2x(x + 2) - 3(x + 2) = 0 \quad \Rightarrow \quad (x + 2)(2x - 3) = 0$$

If $x + 2 = 0$ then $x = -2$

If $2x - 3 = 0$ then $x = \frac{3}{2}$

Hence, the roots of the equation $2x^2 + x - 6 = 0$ are -2 and $\frac{3}{2}$.

$$\begin{aligned} \text{(v)} \quad & 100x^2 - 20x + 1 = 0 \Rightarrow 100x^2 - 10x - 10x + 1 = 0 \\ \Rightarrow & 10x(10x - 1) - 1(10x - 1) = 0 \Rightarrow (10x - 1)(10x - 1) = 0 \\ \Rightarrow & (10x - 1)^2 = 0 \Rightarrow x = \frac{1}{10} \end{aligned}$$

Hence, both the roots of equation $100x^2 - 20x + 1 = 0$ are $\frac{1}{10}, \frac{1}{10}$.

5. Solve for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ ($a \neq 0, b \neq 0, x \neq 0$)

SOLUTION. We have,

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{(a+b)}{ab}$$

(Cancelling $(a + b)$ from both sides)

$$\Rightarrow -\frac{1}{x(a+b+x)} = \frac{1}{ab} \Rightarrow$$

$$-x(a+b+x) = ab$$

$$\Rightarrow x(a+b+x) + ab = 0 \Rightarrow$$

$$x^2 + ax + bx + ab = 0$$

$$\Rightarrow x(x+a) + b(x+a) = 0 \Rightarrow$$

$$(x+a)(x+b) = 0$$

$$\Rightarrow \text{Either } x+a=0 \Rightarrow$$

$$x = -a$$

$$\text{or } x+b=0 \Rightarrow$$

$$x = -b$$

Hence, $x = -a$ and $x = -b$

HOME ASSIGNMENT Ex. 4.2 Q: No 1 to Q2

AHA

- 1. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab .
- 2. If $x = -1/2$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k .

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