QUADRATIC EQUATIONS

PPT5

SUBJECT: MATHEMATICS
CHAPTER NUMBER: 04
CHAPTER NAME : QUADRATIC EQUATIONS

Solution of a Quadratic Equation by Completing the Square

In this method, we convert the quadratic equation into a form so that the term containing x is completely inside a square. Then by taking the square roots, we can easily find its roots.

Steps Involved in the Method of Completing the Square

Step 1 Write the quadratic equation in the form $ax^2 + bx + c = 0$, $a \neq 0$.

Step 2 Divide the equation throughout by a, if it is not unity.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 3 Bring the constant term on R.H.S.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 4 Add square of half the coefficient of x i.e., $\left(\frac{b}{2a}\right)^2$ on both sides.

$$x^2 + 2\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step 5 Write R.H.S. as a perfect square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 6 Take square root of both sides and obtain the values of x.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \ .$$

Hence, $x = \frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{2a}}.$

REMARK Instead of dividing the quadratic equation throughout by a, we can also multiply the equation throughout by a and then complete its square.

LEARNING OUTCOME

1. .Students will be able to solve a Quadratic Equations (factorization & completing the square method).

2. Students will be able to solve real life situations(by forming Quadratic Equations   & solving it by factorization & completing the square method).
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1. Let the present age of Rehman be x years

3 years ago, Rehman's age be $(x - 3)$ years

5 years from now, Rehman's age will be $= (x + 5)$ years

ATQ $\qquad \dfrac{1}{x - 3} + \dfrac{1}{x + 5} = \dfrac{1}{3}$

$\Rightarrow \qquad \dfrac{x + 5 + x - 3}{(x - 3)(x + 5)} = \dfrac{1}{3}$

$\Rightarrow \qquad \dfrac{2x + 2}{x^2 + 2x - 15} = \dfrac{1}{3}$

$\Rightarrow \qquad 6x + 6 = x^2 + 2x - 15$

$\Rightarrow \qquad x^2 - 4x - 21 = 0$

$\Rightarrow \qquad x^2 - 7x + 3x - 21 = 0$

$\Rightarrow \qquad x(x - 7) + 3(x - 7) = 0$

$\Rightarrow \qquad (x + 3)(x - 7) = 0$

$\Rightarrow \qquad x + 3 = 0$ or $x - 7 = 0$

$\Rightarrow x = -3$ (−3 is rejected) or $x = 7$

$\therefore$ Present age of Rehman is 7 years.

2.A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train

2. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train

Total distance travelled = 360 km

Let uniform speed be x km/h then, increased speed be $(x + 5)$ km/h

ATQ $\qquad \dfrac{360}{x} - \dfrac{360}{x + 5} = 1 \quad \left\{ \text{Time} = \dfrac{\text{distance}}{\text{speed}} \right\}$

$\Longrightarrow \qquad \dfrac{360\,(x + 5) - 360x}{x\,(x + 5)} = 1$

$\Longrightarrow \qquad 360x + 1800 - 360x = x\,(x + 5)$

$\Longrightarrow \qquad 1800 = x^2 + 5x$

$\Longrightarrow \qquad x^2 + 5x - 1800 = 0$

$\Longrightarrow \qquad x^2 + 45x - 40x - 1800 = 0$

$\Longrightarrow \qquad x\,(x + 45) - 40\,(x + 45) = 0$

$\Longrightarrow \qquad (x - 40)\,(x + 45) = 0$

$\Longrightarrow \qquad x - 40 = 0 \text{ or } x + 45 = 0$

$\Longrightarrow \qquad x = 40 \text{ or } x = -45 \quad \text{(rejected)}$

$\therefore \quad$ Speed of the train = 40 km/h

3.An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

3.An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Let the speed of passenger train be x km/h then, the speed of express train be $(x + 11)$ km/h

ATQ $\qquad \dfrac{132}{x} - \dfrac{132}{x+11} = 1$

$\Rightarrow \qquad \dfrac{132(x+11) - 132x}{x(x+11)} = 1$

$\Rightarrow \qquad 132x + 1452 - 132x = x^2 + 11x$

$\Rightarrow \qquad x^2 + 11x - 1452 = 0$

$\Rightarrow \qquad x^2 + 44x - 33x - 1452 = 0$

$\Rightarrow \qquad x(x+44) - 33(x+44) = 0$

$\Rightarrow \qquad (x-33)(x+44) = 0$

$\Rightarrow \qquad x - 33 = 0$ or $x + 44 = 0$

$\Rightarrow \qquad x = 33$ or $x = -44$ (rejected)

Hence, speed of the passenger train = 33 km/h

and speed of express train = 33 + 11 = 44 km/h

4. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

4. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank

Let the smaller tap takes x h to fill the tank.

Then, larger tap will take (x − 10)h to fill the same tank.

If the two work together, the amount of water following in one hour $= \frac{1}{x} + \frac{1}{x-10}$

According to the question, we have:

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \quad \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

[∵ The amount of water flowing in 1 h $= \frac{1}{\frac{75}{8}}$]

$\Rightarrow \qquad\qquad 75(2x-10) = 8x(x-10)$

$\Rightarrow \qquad\qquad 150x - 750 = 8x^2 - 80x$

$\Rightarrow \qquad 8x^2 - 230x + 750 = 0$

Here, $a = 8$, $b = -230$ and $c = 750$.

∴ $\quad D = b^2 - 4ac$

$\qquad = (-230)^2 - 4 \times 8 \times 750$

$\qquad = 52900 - 24000 = 28900$

∴ $\qquad x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-(-230) \pm \sqrt{28900}}{2 \times 8}$

$\qquad = \dfrac{230 \pm 170}{16}$

Either $x = \dfrac{230 + 170}{16}$ or $\dfrac{230 - 170}{16}$

$\Rightarrow x = 25$ or $x = \dfrac{15}{4}$

Neglecting $x = \dfrac{15}{4}$, we have $x = 25$.

Hence, the smaller tap takes **25 h** and the larger tap takes **15 h** to fill the tank.

$x = \dfrac{15}{4}$ does not satisfy the condition.

So, $x = \dfrac{15}{4}$ is neglected.

HOME ASSIGNMENT Ex. 4.3 Q. No 4 to Q11

AHA

1. : A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

2 A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.

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