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QUADRATIC EQUATIONS

PPT6

SUBJECT: MATHEMATICS
CHAPTER NUMBER: 04
CHAPTER NAME : QUADRATIC EQUATIONS

PREVIOUS KNOWLEDGE TEST

Solution of a Quadratic Equation by Completing the Square

In this method, we convert the quadratic equation into a form so that the term containing x is completely inside a square. Then by taking the square roots, we can easily find its roots.

Steps Involved in the Method of Completing the Square

Step 1 Write the quadratic equation in the form $ax^2 + bx + c = 0$, $a \neq 0$.

Step 2 Divide the equation throughout by a, if it is not unity.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 3 Bring the constant term on R.H.S.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 4 Add square of half the coefficient of x *i.e.*, $\left(\frac{b}{2a}\right)^2$ on both sides.

$$x^2 + 2\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step 5 Write R.H.S. as a perfect square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 6 Take square root of both sides and obtain the values of x.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}.$$

Hence, $x = \frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{2a}}.$

REMARK Instead of dividing the quadratic equation throughout by a, we can also multiply the equation throughout by a and then complete its square.

LEARNING OUTCOME

1. . Students will be able to solve a Quadratic Equations by quadratic formula

2. Students will be able to solve real life situations (by forming Quadratic Equations
3. Students will be able to find the nature of roots of quadratic equations by quadratic formula.

Discussion on nature of the roots

https://youtu.be/T_7O5VO2ihg(12.20)

Solution of a Quadratic Equation by Using Quadratic Formula

Consider the quadratic equation : $ax^2 + bx + c = 0, \ a \neq 0$

Dividing throughout by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \Rightarrow \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\Rightarrow \quad x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \qquad \left[\text{Adding } \left(\frac{b}{2a}\right)^2 \text{ to both sides}\right]$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow \quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}, \text{ when } b^2 - 4ac \geq 0$$

$$\Rightarrow \quad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is known as quadratic formula or **Shreedharcharay's** formula for finding the roots of a quadratic equation.

Hence, if $b^2 - 4ac \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c$ are given by

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Discriminant. *For the quadratic equation* $ax^2 + bx + c = 0$, *the expression* $D = (b^2 - 4ac)$ *is called its discriminant*. In terms of discriminant D, the two roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

NOTE If the discrimant $D = b^2 - 4ac < 0$, then the quadratic equation $ax^2 + bx + c = 0$ has no real roots.

a quadratic equation $aX^2 + bx + c = 0$ has (i) two distinct real roots, if $b^2 - 4ac > 0$, (ii) two equal roots (i.e., coincident roots), if $b^2 - 4ac = 0$, and (iii) no real roots, if $b^2 - 4ac < 0$

1: Find the roots of the following quadratic equations, if they exist, using the quadratic formula:

(i) $2x^2 – 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $2x^2 + x + 4 = 0$

Solution. (*i*) Given : $2x^2 - 7x + 3 = 0$

Here, $a = 2, b = -7, c = 3$ [On comparing with $ax^2 + bx + c = 0$]

$\therefore$ $D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25 > 0$

So, the given equation has real roots which are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-7) + \sqrt{25}}{2 \times 2} = \frac{7+5}{4} = \frac{12}{4} = 3$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$$

Hence, the roots of the given equation are 3 and $\frac{1}{2}$.

(*ii*) Given : $2x^2 + x - 4 = 0$

Here, $a = 2,\ b = 1,\ c = -4$ [On comparing with $ax^2 + bx + c = 0$]

$\therefore \quad D = b^2 - 4ac = 1^2 - 4 \times 2 \times (-4) = 1 + 32 = 33 > 0$

So, the given equation has real roots which are given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-1+\sqrt{33}}{2\times 2} = \frac{-1+\sqrt{33}}{4}\ ;\ \beta = \frac{-b-\sqrt{D}}{2a} = \frac{-1-\sqrt{33}}{4}$$

Hence, the roots of the given equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

Given : $2x^2 + x + 4 = 0$. Here, $a = 2,\ b = 1,\ c = 4$

$\therefore \quad D = b^2 - 4ac = 1^2 - 4 \times 2 \times 4 = -31 < 0$

As discriminant D is negative, the given quadratic equation has no real roots.

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2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$.

(i) $2x^2 + kx + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where, $a = 2$, $b = k$ and $c = 3$

Discriminant $(D) = b^2 - 4ac$

$= k^2 - 4 \times 2 \times 3$

$= k^2 - 24$

For equal roots, $D = 0$

$\Rightarrow \quad k^2 - 24 = 0$

$\Rightarrow \quad k^2 = 24$ or $k = \pm\sqrt{24}$

$\Rightarrow \quad k = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

$kx(x - 2) + 6 = 0$

$\Rightarrow \quad kx^2 - 2kx + 6 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = k$, $b = -2k$ and $c = 6$

Discriminant $(D) = b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 6$

$= 4k^2 - 24k$

For equal roots, $D = 0$

$\Rightarrow \quad 4k^2 - 24k = 0$

$\Rightarrow \quad k(4k - 24) = 0$

$\Rightarrow$ $k = 0$ (not possible) or $4k - 24 = 0$

$\Rightarrow \quad 4k = 24 \quad \Rightarrow \quad k = \frac{24}{4} = 6$

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3.Find the values of P for which the following quadratic equation has two equal roots $Px(x-3)+9=0$

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. Find the value of p so that the quadratic equation $px(x-3)+9=0$ has two equal roots.

Sol. $$px(x-3)+9=0$$

$\Rightarrow$ $$px^2-3px+9=0.$$

Here, $a=p,\ b=-3p,\ c=9$

For equal roots $D=0$

$\Rightarrow$ $$D=b^2-4ac$$

$\Rightarrow$ $$(-3p)^2-4\times p\times 9=0$$

$\Rightarrow$ $$9p^2-36p=0$$

$\Rightarrow$ $$9p(p-4)=0$$

$\Rightarrow$ $9p=0$ or $p-4=0$

$\Rightarrow$ $p=0$ or $p=4$

but $p\neq 0$ [$\because$ In quadratic equation $a\neq 0$] $\therefore p=4$

HOME ASSIGNMENT Ex. 4.4.

AHA

1. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

2. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m . Find its length and breadth.

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