QUADRATIC EQUATIONS

PPT7

SUBJECT: MATHEMATICS
CHAPTER NUMBER: 04
CHAPTER NAME : QUADRATIC EQUATIONS

Website: www.odmegroup.org
Email: info@odmps.org

Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

ODM EDUCATIONAL GROUP
Changing your Tomorrow

1.The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2+bx+c=0$, where a,b and c are real numbers and $a\neq0$.
'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term..

2.For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the discriminant, (denoted by D), of the quadratic equation.

3.The discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic equation.

4.Nature of Roots

Based on the value of the discriminant, $D=b^2-4ac$, the roots of a quadratic equation can be of three types.

Case 1: If $D>0$, the equation has two distinct real roots.

Case 2: If $D=0$, the equation has two equal real roots.

Case 3: If $D<0$, the equation has no real roots.

LEARNING OUTCOME

1. . Students will be able to solve a Quadratic Equations by quadratic formula

2. Students will be able to solve real life situations (by forming Quadratic Equations)
3. Students will be able to find the nature of roots of quadratic equations by quadratic formula.
4. .Students will be able to find solution of quadratic equation by completing the square method.

1.Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(B) $-2\ x^2 = (5 - x)\ (2x - 25\ )$

(C) $(k+ 1)\ x^2 + 32\ x = 7$,where $k = -1$

(D) $x^3 - x^2 = (x - 1\ )$

1.Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(B) $-2\ x^2 = (5 - x)\ (2x - 25\ )$

(C) $(k+ 1)\ x^2 + 32\ x = 7$,where $k = -1$

(D) $x^3 - x^2 = (x - 1\ )$

A)Given equation is, $x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$

$\Rightarrow 10x - 18 = 0$

which is not of the form $ax^2 + bx + c$, $a \neq 0$. Thus, the equation is not a quadratic equation.

B) Given equation is $-2\ x^2 = (5 - x)\ (2x - 25\ )$

$\Rightarrow -2\ x^2 = 10x - 2\ x^2 - 2 + 2x5$

$\Rightarrow 0 = 10x - 2 + 2x5$

$\Rightarrow 50x + 2x - 10 = 0$

$\Rightarrow 52x - 10 = 0$

which is also not a quadratic equation

C)Given equation is, x^2 (k +1) + 32 x = 7

$\because$ k = -1 [Given]

$\Rightarrow x^2 (-1 + 1) + 32x = 7$

$\Rightarrow 32x = 7$

$\Rightarrow 3x - 14 = 0$

which is also not a quadratic equation.

d)Given equation is, $x^3 - x^2 = (x - 1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2(1) + 3x(1)^2 - (1)^3$

[$\because (a - b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b$]

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

which represents a quadratic equation because it is of the quadratic form $ax^2 + bx + c = 0, a \neq 0$

2. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$

(B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$

(D) $3x^2 - 6x - 2 = 0$

2. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$

(B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$

(D) $3x^2 - 6x - 2 = 0$

A)Substituting $x = 2$ in $x^2 - 4x + 5$, we get

$(2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1 \neq 0$

So, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$

B) Substituting $x = 2$ in $x^2 + 3x - 12$ we get

$(2)^2 + 3(2) - 12 = 4 + 6 - 12 = -2 \neq 0$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$

(C) Substituting $x = 2$ in $2x^2 - 7x + 6$, we get $2(2)^2 - 7(2) + 6$

$= 8 - 14 + 6 = 14 - 14 = 0$

So, $x = 2$ is a root of $2x^2 - 7x + 6 = 0$

(D) Substituting $x = 2$ in $3x^2 - 6x - 2$, we get

$3(2)^2 - 6(2) - 2 = 12 - 12 - 2 = -2 \neq 0$

So, $x = 2$ is not a root of $3x^2 - 6x - 2 = 0$

3. If 1/2 is a root of the equation x^2 + kx – 54 = 0, then the value of k is

(A) 2

(B) -2

(C) 1/4

(D) 1/2

3. If 1/2 is a root of the equation x^2 + kx − 54 = 0, then the value of k is

(A) 2

(B) -2

(C) 1/4

(D) ½

Since, 1/2 is a root of the quadratic equation x^2 + kx – 54 = 0

$$\therefore \quad \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0 \Rightarrow \frac{1+2k-5}{4} = 0$$

$$\Rightarrow \quad 2k - 4 = 0$$

$$\Rightarrow \quad 2k = 4 \Rightarrow \quad k = 2$$

4. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots

(B) two equal real roots

(C) no real roots

(D) more than 2 real roots

.

4 .The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots

(B) two equal real roots

(C) no real roots

(D) more than 2 real roots

Solution:

(C) Given equation is $2x^2 - \sqrt{5}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 2$,$b = -77$ and $c = 1$

$D = b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1) = 5 - 8$

$= -3 < 0$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots.

5. Which of the following equations has no real roots?

(A) $x^2 - 4x + 3\sqrt{2} = 0$

(B) $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$

(D) $3x^2 + 4\sqrt{3}x + 4 = 0$

5. Which of the following equations has no real roots?

(A) $x^2 - 4x + 3\sqrt{2} = 0$

(B) $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$

(D) $3x^2 + 4\sqrt{3}x + 4 = 0$

A) given equation is $x^2 - 4x + 3\sqrt{2} = 0$

On comparing with $ax^2 + bx + c - 0$, we get

$a = 1$, $b = -4$ and $c = 3\sqrt{2}$

The discriminant of $x^2 - 4x + 3\sqrt{2} = 0$ is

$D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2}$

$= 16 - 12 \times (1.41)$

$= 16 - 16.92$

$= -0.92$

$\because b^2 - 4ac < 0$

$\therefore$ Roots of the equation are not real.

(B) Given equation is $x^2 + 4x - 3\sqrt{2} = 0$
On comparing with $ax^2 + bx + c = 0$,
we get
$a = 1, b = 4$ and $c = -3\sqrt{2}$
$\therefore D = b^2 - 4ac = (4)^2 - 4(1)(-3\sqrt{2})$
$= 16 + 12\sqrt{2} > 0$
Hence, the equation has real roots
(C) Given equation is $x^2 - 4x - 3\sqrt{2} = 0$
On comparing with $ax^2 + bx + c = 0$,
we get $a = 1$, $b = -4$ and $c = -3\sqrt{2}$
$\therefore D = b^2 - 4ac = (-4)^2 - 4(1)(-3\sqrt{2})$
$= 16 + 12\sqrt{2} > 0$
Hence, the equation has real roots
(D) Given equation is $3x^2 + 4\sqrt{3}x + 4 = 0$
On comparing with $ax^2 + bx + c = 0$, we get
$a = 3, b = 4\sqrt{3}$ and $c = 4$
$\therefore D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4)$
$= 48 - 48 = 0$
Hence, the equation has real and equal roots

6. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2 (1 + m^2)$.

The given equation is $(1 + m^2) x^2 + (2mc) x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore $D = 0$

$\Rightarrow B^2 - 4AC = 0.$

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\Rightarrow m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0.$ [Dividing throughout by 4]

$\Rightarrow -c^2 + a^2 (1 + m^2) = 0$

$\Rightarrow c^2 = a^2 (1 + m^2).$

Hence Proved

HOME ASSIGNMENT Ex. 4.1.Q1 to 10(EXEMPLAR)

AHA

1.Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer

THANKING YOU

ODM EDUCATIONAL GROUP