EDUCATIONAL GROUP
Changing your Tomorrow

QUADRATIC EQUATIONS

PPT8

SUBJECT: MATHEMATICS
CHAPTER NUMBER: 04
CHAPTER NAME : QUADRATIC EQUATIONS

CHANGING YOUR TOMORROW

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PREVIOUS KNOWLEDGE TEST

1.The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2+bx+c=0$, where a,b and c are real numbers and $a \neq 0$. 'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term..

2.For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the discriminant, (denoted by D), of the quadratic equation.

3.The discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic equation.

4.Nature of Roots

Based on the value of the discriminant, $D=b^2-4ac$, the roots of a quadratic equation can be of three types.

Case 1: If $D>0$, the equation has two distinct real roots.

Case 2: If $D=0$, the equation has two equal real roots.

Case 3: If $D<0$, the equation has no real roots.

LEARNING OUTCOME

1. . Students will be able to solve a Quadratic Equations by quadratic formula

2. Students will be able to solve real life situations (by forming Quadratic Equations)
3. Students will be able to find the nature of roots of quadratic equations by quadratic formula.
4. .Students will be able to find solution of quadratic equation by completing the square method.

- Short Tricks to solve quadratic equations and word sums.

https://youtu.be/GxUVVOb3ywE)

- : How to solve quadratic equations in completing the square method
- https://youtu.be/Eob9SOf5DzQ

1.If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the value of k.

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Since – 5 is a root of the equation $2x^2 + px - 15 = 0$

$\therefore 2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0$

or $5p = 35$

or $p = 7$

Again $p(x^2 + x) + k = 0$

or $7x^2 + 7x + k = 0$ has equal roots

$a = 7$, $b = 7$, $c = k$

$\therefore D = 0$

i.e., $b^2 - 4ac = 0$ or $49 - 4 \times 7k = 0$

$\Rightarrow k = \frac{49}{28} = \frac{7}{4}$.

2.Using quadratic formula solve the following quadratic equation:

$p^2x^2 + (p^2 - q^2)\,x - q^2 = 0$

2.Using quadratic formula solve the following quadratic equation:
$p^2x^2 + (p^2 - q^2) x - q^2 = 0$
Solution:
We have $p^2x^2 + (p^2 - q^2) x - q^2 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we have
$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$
$\therefore D = b^2 - 4ac$
$\Rightarrow (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$
$\Rightarrow (p^2 - q^2)^2 + 4p^2q^2$
$\Rightarrow (p^2 + q^2)^2 > 0$
So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

3.If the roots of the quadratic equation $(a - b) x^2 + (b - c) x + (c - a) = 0$ are equal, prove that $2a = b + c$..

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Since the equation $(a - b) x^2 + (b - c) x + (c - a) = 0$ has equal roots, therefore discriminant

$D = (b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$

$\Rightarrow b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

$\Rightarrow (2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)2a = 0$

$\Rightarrow (2a - b - c)^2 = 0$

$\Rightarrow 2a - b - c = 0$

$\Rightarrow 2a = b + c.$

Hence Proved

4.If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2 (1 + m^2)$.

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The given equation is $(1 + m^2) x^2 + (2mc) x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore $D = 0$

$\Rightarrow B^2 - 4AC = 0.$

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\Rightarrow m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0.$ [Dividing throughout by 4]

$\Rightarrow -c^2 + a^2 (1 + m^2) = 0$

$\Rightarrow c^2 = a^2 (1 + m^2).$

Hence Proved

5.The difference of two natural numbers is 5 and the difference of their reciprocals is 1/10. Find the numbers.

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Let the two natural numbers be x and y such that x > y.
According to the question,
Difference of numbers, x – y = 5 ⇒ x = 5 + y .....(i)
Difference of the reciprocals

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \qquad ...(ii)$$

Putting the value of (i) in (ii)

$$\frac{1}{y} - \frac{1}{5+y} = \frac{1}{10} \qquad \Rightarrow \frac{5+y-y}{y(5+y)} = \frac{1}{10}$$

$$\Rightarrow \quad 50 = 5y + y^2 \qquad \Rightarrow \quad y^2 + 5y - 50 = 0$$

$$\Rightarrow \quad y^2 + 10y - 5y - 50 = 0 \qquad \Rightarrow \quad y(y + 10) - 5(y + 10) = 0$$

$$\Rightarrow \quad (y - 5)(y + 10) = 0$$

∴ y is a natural number.
∵ y = 5
Putting the value of y in (i), we have
⇒ x = 5 + 5
⇒ x = 10
The required numbers are 10 and 5.

HOME ASSIGNMENT Ex. 4.1.Q1 to 10(EXEMPLAR)
AHA

1. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

2. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m . Find its length and breadth.

THANKING YOU

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