ARITHMETIC PROGRESSOINS
PPT-7

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME : ARITHMETIC PROGRESSIONS

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PREVIOUS KNOWLEDGE TEST

- The nth term an of the AP with first term a and common difference d is given by nth term a_n = a + (n – 1) d.

- a_n is also called the general term of the AP.
- nth term of an AP from the end = l – (n – 1) d
- Sum of the first n terms of AP; $s_n = \frac{n}{2}[2a + (n-1)d]$
- OR
- $S_n = \frac{n}{2}(a + a_n)$
- SELECTION OF TERMS;
- For 3 numbers in AP (a-d),a,(a+d)

- For 4 numbers in AP (a-3d),(a-d),(a+d),(a+3d)

LEARNING OUTCOME

1. Students will be able to know AP as list of numbers in which successive term is obtained by adding a fixed number to previous term.
2. Students will be able to observe geometric patterns and understand the concept of AP
3. Students will be able to identify situations in daily life where the AP is observed.
4. Students will be able to identify the first term & common difference.
5. Students will be able to calculate the required term.
6. Students will be able to find nth term from the end of the AP.

- Short Tricks to solve AP.
- https://youtu.be/0p_yi9WXjBs

1.Find the sum of last ten terms of the AP: 8,10, 12,……., 126.

Solution. Given the AP : 8, 10, 12, ..., 126.

Here, $a = 8$, $d = 2$, $a_n = 126$ (say)

$$\Rightarrow \quad a + (n-1)d = 126$$

$$\Rightarrow \quad 8 + (n-1) \times 2 = 126 \quad \Rightarrow (n-1) \times 2 = 118 \quad \Rightarrow n - 1 = 59$$

$$\therefore \quad n = 60$$

Clearly, the last 10 terms of the AP will be from 51st term to 60th term, which also form an AP.

$$a_{51} = a + 50d = 8 + 50 \times 2 = 108$$

Also, $a_{60} = 126$

Hence, the sum of last ten terms of the given AP will be

$$S = \frac{10}{2}[a_{51} + a_{60}] = 5(108 + 126) = 5 \times 234 = 1170.$$

2.If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289,find the sum of first n terms

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Given, $S_7 = 49$

$\Rightarrow \quad 49 = \frac{7}{2}[2a+(7-1)\times d] \quad \Rightarrow 7\times 2 = [2a+6d]$

$\Rightarrow \quad 14 = 2a+6d \quad \Rightarrow a+3d = 7 \quad ...(1)$

and $S_{17} = 289$

$\Rightarrow \quad 289 = \frac{17}{2}[2a+(17-1)\,d] \quad \Rightarrow 2a+16d = \frac{289\times 2}{17} = 34$

$\Rightarrow \quad a+8d = 17 \quad ...(2)$

Now subtracting equation (1) from (2), we get

$5d = 10 \quad \Rightarrow \quad d = 2$

Putting the value of d in equation (1), we get

$a+3\times 2 = 7 \quad \Rightarrow \quad a = 7-6 = 1$

Here $a = 1$ and $d = 2$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2\times 1+(n-1)\times 2] = \frac{n}{2}[2+2n-2] = \frac{n}{2}\times 2n = n^2$

3.The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161.Find the 28^{th} term of this AP.

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Solution. Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_7 = \frac{7}{2}[2a + (7-1)d]$ $\Rightarrow S_7 = \frac{7}{2}[2a + 6d]$

$\Rightarrow$ $63 = 7a + 21d$ $[S_7 = 63 \text{ (given)}]$

$\Rightarrow$ $a = \frac{63 - 21d}{7}$ ...(1)

Also, $S_{14} = \frac{14}{2}[2a + 13d]$

$\Rightarrow$ $S_{14} = 14a + 91d$

But according to question, $S_{1-7} + S_{8-14} = S_{14}$

$\Rightarrow$ $63 + 161 = 14a + 91d$ $\Rightarrow 224 = 14a + 91d$

$\Rightarrow$ $2a + 13d = 32$ $\Rightarrow 2\left(\frac{63 - 21d}{7}\right) + 13d = 32$ ...(2)

$\Rightarrow$ $126 - 42d + 91d = 224$

$\Rightarrow$ $49d = 98 \Rightarrow d = 2$

$\therefore$ $a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = \frac{21}{7} = 3$

Thus, $a_{28} = a + 27d = 3 + 27 \times 2$

$\Rightarrow$ $a_{28} = 3 + 54 = 57$

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$$S_{30} = 3(S_{20} - S_{10})$$

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Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{30} = \frac{30}{2}[2a + (30-1)d] \Rightarrow S_{30} = \frac{30}{2}[2a + 29d]$

$\Rightarrow$ $S_{30} = 15(2a + 29d) = 30a + 435d$ ...(1)

and $S_{20} = \frac{20}{2}[2a + (20-1)d] = \frac{20}{2}[2a + 19d]$

$S_{20} = 10(2a + 19d) = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + (10-1)d] \Rightarrow \frac{10}{2}[2a + 9d]$

$\Rightarrow$ $S_{10} = 5(2a + 9d) = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30}$ [From (1)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

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Given, $$S_n = 3n^2 - 4n \quad ...(1)$$

Replacing n by $n - 1$, we get

$$S_{n-1} = 3\,(n-1)^2 - 4\,(n-1) \quad ...(2)$$

Since,

$$a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$$
$$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$$
$$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$$

So, nth term, $a_n = 6n - 7 \quad ...(3)$

Substituting $n = 1, 2, 3, ...$ respectively in (3), we get

$$a_1 = 6 \times 1 - 7 = -1, a_2 = 6 \times 2 - 7 = 5$$

and $$a_3 = 6 \times 3 - 7 = 11$$

Hence, AP is $-1, 5, 11, ...$

12th term, $a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$ [From (3)]

HOME ASSIGNMENT Ch -5

THANKING YOU

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