ARITHMETIC PROGRESSOINS

INTRODUCTION

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME :ARITHMETIC PROGRESSIONS

LEARNING OUTCOME

1. Students will be able to know AP as list of numbers in which successive term is obtained by adding a fixed number to previous term.
2. Students will be able to observe geometric patterns and know the concept of AP
3. Students will be able to identify situations in daily life where the AP is observed.
4. Students will be able to identify the first term & common difference.
5. Students will be able to calculate the required term.

Introduction to AP ;

https://youtu.be/dv3lcYhxZEM (9.53)

- An arrangement of numbers in a definite order according to some rule is called a Sequence.
- . In other words, a pattern of numbers in which succeeding terms are obtained from the preceding term by adding/subtracting a fixed number or by multiplying with/dividing by a fixed number, is called sequence or list of numbers.
  e.g. 1,2,3,4,5

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- This fixed number is called the common difference of the AP. It can be positive, negative or zero.
- Let us denote the first term of an AP by a_1, second term by a_2, . . ., nth term by a a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, . . ., a_n$ So, $a_2 - a_1 = a_3 - a_2 = . . . = a_n - a_{n-1} = d$.
- The general form of an arithmetic progression is given by
  $a, a + d, a + 2d, a + 3d, . . .$ where a is the first term and d the common difference.

: .In the following situation, does the list of numbers involved make an arithmetic progression, and why?

(i)   The taxi fare after each km when the fare is ` 15 for the first km and ` 8 for each additional km.

(ii)   The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

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.

(i) Given, $a_1 = $ ₹ 15, $a_2 = $ ₹ 15 + ₹ 8 = ₹ 23,

$a_3 = $ ₹ 23 + ₹ 8 = ₹ 31

∴ List of fares is ₹ 15, ₹ 23, ₹ 31 .....

Now, $a_2 - a_1 = $ ₹ 23 − ₹ 15 = ₹ 8

$a_3 - a_2 = $ ₹ 31 − ₹ 23 = ₹ 8

Here, $a_2 - a_1 = a_3 - a_2$

Thus, the list forms an AP.

(ii) Let $a_1 = x; \quad a_2 = x - \frac{1}{4}x = \frac{3}{4}x;$

$a_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right)$

$= \frac{3}{4}x - \frac{3}{16}x = \frac{9}{16}x$

The list of numbers is $x, \frac{3}{4}x, \frac{9}{16}x$ .....

$a_2 - a_1 = \frac{3}{4}x - x = -\frac{1}{4}x;$

$a_3 - a_2 = \frac{9}{16}x - \frac{3}{4}x = -\frac{3x}{16}$

Here, $a_2 - a_1 \neq a_3 - a_2$

∴ It is not an AP.

Which of the following list of numbers form an AP? If they form an AP, write the next two terms :
 (i) 2, 4, 8, 16, . . .
(ii) 2, 5/2, 3, 7/2, ...

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(i) 2, 4, 8, 16, . . .
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(i) 2, 4, 8, 16,...

Here, $\qquad a_2 - a_1 = 4 - 2 = 2 ; \quad a_3 - a_2 = 8 - 4 = 4$

$\Rightarrow \qquad a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP.

(ii) $2, \dfrac{5}{2}, 3, \dfrac{7}{2}$

Here, $\quad a_2 - a_1 = \dfrac{5}{2} - 2 = \dfrac{5-4}{2} = \dfrac{1}{2} ; \quad a_3 - a_2 = 3 - \dfrac{5}{2} = \dfrac{6-5}{2} = \dfrac{1}{2} ; \quad a_4 - a_3 = \dfrac{7}{2} - 3 = \dfrac{7-6}{2} = \dfrac{1}{2}$

$\Rightarrow \qquad a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow \qquad a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = \dfrac{1}{2}$.

The next three terms are :

$a_5 = a_4 + d = \dfrac{7}{2} + \dfrac{1}{2} = 4, \qquad a_6 = a_5 + d = 4 + \dfrac{1}{2} = \dfrac{9}{2}, \qquad a_7 = a_6 + d = \dfrac{9}{2} + \dfrac{1}{2} = 5.$

Write first four terms of the AP, when the first term a and the common difference d are given as follows:
(i) a = 10, d = 10
(ii) a = −2, d = 0
(iii) a = 4, d = − 3, ...

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(i) a = 10, d = 10
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(iii) a = 4, d = − 3

(i) Given, $a = 10, d = 10$
$a_1 = 10, a_2 = 10 + 10 = 20$
$a_3 = 20 + 10 = 30;$
$a_4 = 30 + 10 = 40$

Thus, the first four terms of the AP are 10, 20, 30 and 40 respectively.

(ii) Given, $a = -2, d = 0$
The first four terms of the AP are −2, −2, −2 and −2.

(iii) $a_1 = 4, d = -3$
and $a_2 = 4 + d = 4 - 3 = 1$
$a_3 = 1 + d = 1 - 3 = -2$
and $a_4 = -2 + d = -2 - 3 = -5$
∴ The first four terms are 4, 1, − 2 and − 5.

Which of the following list of numbers form an AP? If they form an AP, write the next two terms :
 (i)-10,-6,-2,2,……

(ii)$\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, …

4.Which of the following list of numbers form an AP? If they form an AP, write the next two terms :

(iv)-10,-6,-2,2,…… (viii)$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, …$

(iv) $-10, -6, -2, 2, …$

$$a_2 - a_1 = -6 - (-10) = 4$$

and $\quad a_3 - a_2 = -2 - (-6) = 4$

$$a_3 - a_2 = a_2 - a_1$$

∴ The given sequence is an AP.

Here, $a_1 = a = -10, d = 4$

∴ $\quad a_5 = 2 + 4 = 6 ; a_6 = 6 + 4 = 10 ;$

$$a_7 = 10 + 4 = 14$$

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ……$

Here, $\quad a_2 - a_1 = \sqrt{8} - \sqrt{2}$

$$= 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

and $\quad a_3 - a_2 = \sqrt{18} - \sqrt{8}$

$$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

$$a_3 - a_2 = a_2 - a_1$$

∴ The given sequence is an AP.

Next three terms are

$$a_5 = \sqrt{32} + d = \sqrt{16 \times 2} + d$$

$$= 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$$

$$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$$

and $\quad a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

HOME ASSIGNMENT Ex. 5.1 Q. No 1 to Q4

AHA

- Which of the following are APs? If they form an AP, find the common difference d and write three more terms
  - (i) 2, 4, 8, 16, ... (ii) – 1.2, – 3.2, – 5.2, – 7.2, . . .

THANKING YOU
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