

ARITHMETIC PROGRESSIONS

PPT-4

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME : ARITHMETIC PROGRESSIONS

CHANGING YOUR TOMORROW

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PREVIOUS KNOWLEDGE TEST

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- The general form of an arithmetic progression is given by $a, a + d, a + 2d, a + 3d, \dots$ where a is the first term and d the common difference.

nth term $a_n = a + (n - 1) d$.

So, the nth term of the AP with first term a and common difference d is given by

- a_n is also called the general term of the AP.
- nth term of an AP from the end = $l - (n - 1) d$

LEARNING OUTCOME

1. Students will be able to know the sum of first n terms of an AP.
2. Students will be able to solve problems based on finding the number of terms, when sum of terms and AP are given.

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Sum of first n-terms of an AP
<https://youtu.be/cRyeMph7tKU>(12.52)

1. Find the sum of the following AP:

2, 7, 12, . . . , to 10 terms.

2, 7, 12,, to 10 terms

Here, $a = 2$, $d = 7 - 2 = 5$, $n = 10$, $S_{10} = ?$

$$\therefore S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$\begin{aligned} \therefore S_{10} &= \frac{10}{2} [2 \times 2 + (10 - 1) 5] \\ &= 5 [4 + 45] = 5 \times 49 = 245 \end{aligned}$$

2. Find the sum $34 + 32 + 30 + \dots + 10$

$$34 + 32 + 30 + \dots + 10$$

Here, $a = 34, d = 32 - 34 = -2$

$$a_n = 10$$

$$a_n = a + (n - 1) d$$

$$\Rightarrow 10 = 34 + (n - 1) (-2)$$

$$\Rightarrow 10 - 34 = (n - 1) (-2)$$

$$\Rightarrow \frac{-24}{-2} = n - 1 \Rightarrow n - 1 = 12$$

$$\Rightarrow n = 12 + 1 = 13$$

$$\therefore S_{13} = \frac{13}{2} [34 + 10]$$

$$= \frac{13}{2} \times 44 = 13 \times 22 = 286$$

3. In an AP: given $a = 5$, $d = 3$, $a_n = 50$, find n .

$$a = 5, d = 3, a_n = 50$$

$$a_n = 50$$

$$\Rightarrow a + (n - 1) d = 50$$

$$\Rightarrow 5 + (n - 1) 3 = 50$$

$$\Rightarrow (n - 1) 3 = 50 - 5$$

$$\Rightarrow (n - 1) 3 = 45$$

$$\Rightarrow n - 1 = \frac{45}{3} = 15 \Rightarrow n = 16$$

4. given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} .

4. given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} .

$$\text{Given, } a_3 = 15, S_{10} = 125$$

$$a_3 = 15$$

$$\Rightarrow a + 2d = 15$$

$$\Rightarrow a = 15 - 2d$$

$$\text{We have, } S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$S_{10} = \frac{10}{2} [2a + (n - 1) d]$$

$$= 5 [2 (15 - 2d) + (10 - 1) \times d]$$

$$= 5 [30 - 4d + 9d]$$

$$\Rightarrow 125 = 5 (30 + 5d)$$

$$\Rightarrow \frac{125}{5} = 30 + 5d$$

$$\Rightarrow 25 - 30 = 5d$$

$$\Rightarrow -5 = 5d$$

$$\Rightarrow \frac{-5}{5} = d$$

$$\Rightarrow d = -1$$

$$a_{10} = a + 9d = 15 - 2d + 9d$$

$$= 15 + 7d = 15 + 7 \times (-1)$$

$$= 15 - 7 = 8$$

5. given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a .

5. given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a .

$$\begin{aligned} a_n &= 4, d = 2, S_n = -14 \\ S_n &= \frac{n}{2}[a + a_n] \\ \Rightarrow -14 &= \frac{n}{2}[a + 4] \\ \Rightarrow -28 &= na + 4n \\ \Rightarrow \frac{-28 - 4n}{n} &= a \\ a_n &= 4 \\ \Rightarrow a + (n - 1)d &= 4 \\ \Rightarrow \frac{-4n - 28}{n} + (n - 1)2 &= 4 \\ \Rightarrow -4n - 28 + 2n^2 - 2n &= 4n \\ \Rightarrow 2n^2 - 10n - 28 &= 0 \text{ or } n^2 - 5n - 14 = 0 \\ \text{or } n^2 - 7n + 2n - 14 &= 0 \\ \Rightarrow n(n - 7) + 2(n - 7) &= 0 \\ \Rightarrow (n + 2)(n - 7) &= 0 \\ \Rightarrow \text{Either } n + 2 = 0 \text{ or } n - 7 &= 0 \\ \Rightarrow n = -2 \text{ or } n = 7, n = -2 &\text{ is rejected} \\ \therefore a &= \frac{-28 - 4n}{n} = \frac{-28 - 4 \times 7}{7} \\ &= \frac{-28 - 28}{7} = \frac{-56}{7} = -8 \end{aligned}$$

6. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

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Sol. Given, $d = 7$, $a_{22} = 149$

$$a_{22} = a + 21d = 149$$

$$\Rightarrow a + 21 \times 7 = 149$$

$$\Rightarrow a + 147 = 149$$

$$\Rightarrow a = 149 - 147 = 2$$

$$\begin{aligned} \therefore S_{22} &= \frac{22}{2} [a + a_{22}] \\ &= 11 [2 + 149] \\ &= 11 \times 151 = 1661 \end{aligned}$$

7. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?.

How many terms of AP: 9, 17, 25, must be taken to give a sum of 636?

Sol. Given, $a = 9$, $d = 17 - 9 = 8$,

$$S_n = 636$$

$$S_n = \frac{n}{2}[2a + (n - 1) d]$$

$$\Rightarrow 636 = \frac{n}{2}[2 \times 9 + (n - 1) 8]$$

$$\Rightarrow 636 \times 2 = n [18 + 8n - 8]$$

$$\Rightarrow 636 \times 2 = n (10 + 8n)$$

$$\Rightarrow 636 \times 2 = 2n(5 + 4n)$$

$$\Rightarrow \frac{636 \times 2}{2} = 5n + 4n^2$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow 4n(n + 53) - 48(n + 53) = 0$$

$$\Rightarrow (4n - 48)(n + 53) = 0$$

$$\Rightarrow \text{Either } 4n = 48 \text{ or } n + 53 = 0$$

$$\Rightarrow n = \frac{48}{4} \text{ or } n = -53 \text{ (rejected)}$$

$$\Rightarrow n = 12$$

Hence $n = 12$

HOME ASSIGNMENT Ex. 5.3 Q: No 1 to Q10

AHA

1. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?
2. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

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