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Changing your Tomorrow

ARITHMETIC PROGRESSOINS
PPT-5

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME : ARITHMETIC PROGRESSIONS

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PREVIOUS KNOWLEDGE TEST

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding afixed number to the preceding term except the first term.
- The general form of an arithmetic progression is given by
  a, $a + d$, $a + 2d$,$a + 3d$, . . .where a is the first term and d the common difference.

  nth term a_n = a + (n – 1) d.
  So, the nth term an of the AP with first term a and common difference d is given by
  - a_n is also called the general term of the AP.
  - nth term of an AP from the end = l – (n – 1) d
  - Sum of the first n terms of AP;$s_n = \frac{n}{2}[2a + (n-1)d]$ OR
  - $$S_n = \frac{n}{2}(a + a_n)$$

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LEARNING OUTCOME

1. Students will be able to solve problems based on finding the nth term, when sum of first nth terms is given.
2. Students will be able to solve problems based on finding the number of terms, when sum of terms and AP are given.
3. Students will be able to solve problems based on finding sum of first nth terms when the nth term is given.

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Problems based on Sum of first n-terms of an AP
https://youtu.be/AJwACuH9j5M (14.35)

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1.Find the sum of first 15 terms of the list of numbers whose nth term is given by $a_n = 9 - 5n$.

$a_n = 9 - 5n$

Putting $n = 1, 2, 3, ...$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$d = a_2 - a_1 = -1 - 4 = -5$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$d = a_3 - a_2 = -6 - (-1)$

$= -6 + 1 = -5$

$\therefore$ The sequence $4, -1, -6, ...$ is an A.P.

Here, $a = 4, d = -1 - 4 = -5$

$$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)]$$

$$= \frac{15}{2}[8 - 70] = \frac{15}{2} \times (-62)$$

$= -465$

2.If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Sol. $S_n = 4n - n^2$

Putting, $n = 1, 2, 3, \ldots$

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$a_1 = 3$

$S_2 = 4\,(2) - 2^2 = 8 - 4 = 4$

$a_1 + a_2 = 4$

$\Rightarrow \quad 3 + a_2 = 4$

$\Rightarrow \quad a_2 = 4 - 3 = 1$

$d = a_2 - a_1 = 1 - 3 = -2$

$a_3 = a_2 + d = 1 + (-2) = -1$

$a_{10} = a + 9d = 3 + 9\,(-2)$

$= 3 - 18 = -15$

$a_n = a + (n - 1)\,d$

$= 3 + (n - 1)\,(-2)$

$= 3 - 2n + 2 = 5 - 2n$

3. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: RS 200 for the first day, RS` 250 for the second day, RS 300 for the third day, etc., the penalty for each succeeding day being `RS 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Since the penalty for each succeeding day is Rs *50* more than for the preceding day. Therefore, amount of penalty for different days forms an A.P. with first term a (= 200) and common difference d (= 50). We have to find how much does a delay of 30 days cost the contractor? In other words, we have to find the sum of 30 terms of the A.P.

$\therefore$ Required sum $= \frac{30}{2}\{2 \times 200 + (30 - 1) \times 50\}$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ Required sum $= 15\,(400 + 29 \times 50)$

$\Rightarrow$ Required sum $= 15\,(400 + 1450)$

$\Rightarrow$ Required sum $= 15 \times 1850 = 27750$

Thus, a delay of 30 days will cost the contractor of Rs 27750.

4. A sum of RS 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is RS 20 less than its preceding prize, find the value of each of the prizes

A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let 1st prize be of ₹ a; therefore, 2nd prize be ₹ $(a-20)$ and 3rd prize be ₹ $(a-20-20) =$ ₹ $(a-40)$

Then seven prizes are

₹ a, ₹ $(a-20)$, ₹ $(a-40)$, ... to 7 terms $= 700$

Here, $a_1 = a$, $d =$ ₹ $(a-20-a) =$ − ₹ 20

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_7 = \frac{7}{2}[2a + (7-1)\,d]$$

$$\Rightarrow \quad 700 = \frac{7}{2}[2a + 6d]$$

$$\Rightarrow \quad 700 = \frac{7}{2}[2a + 6(-20)]$$

$$\Rightarrow \quad 700 \times \frac{2}{7} = 2a - 120$$

$$\Rightarrow \quad 200 + 120 = 2a$$

$$\Rightarrow \quad 320 = 2a$$

$$\Rightarrow \quad a = \frac{320}{2} = ₹\ 160$$

So, the seven prizes are ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

5. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?.

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Let the trees be planted 1, 2, 3, 4, 5 ....., 12

Here, $a = 1, d = 1, n = 12$

Total number of trees planted by one section

$$S_{12} = \frac{12}{2}[2a + (n - 1)\, d]$$

$$= 6\,[2 \times 1 + (12 - 1) \times 1]$$

$$= 6\,[2 + 11] = 6 \times 13 = 78$$

Total number of trees planted by 3 sections

$= 78 \times 3 = 234$

HOME ASSIGNMENT Ex. 5.3 Q. No 11 to Q20

AHA

1. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

2. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

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