ODM EDUCATIONAL GROUP
Changing your Tomorrow

ARITHMETIC PROGRESSOINS
PPT-6

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME : ARITHMETIC PROGRESSIONS

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PREVIOUS KNOWLEDGE TEST

- The nth term an of the AP with first term a and common difference d is given by nth term a_n = a + (n – 1) d.

- a_n is also called the general term of the AP.
- nth term of an AP from the end = l – (n – 1) d
- Sum of the first n terms of AP; $s_n = \frac{n}{2}[2a + (n-1)d]$
- OR
- $$S_n = \frac{n}{2}(a + a_n)$$

LEARNING OUTCOME

1. Students will be able to solve questions involving nth term and sum of the first nth terms.
2. Students will be able to solve questions involving selection of terms.
3. Students will be able to solve problems based on finding sum of first nth terms when the nth term is given.

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Selection of terms

For convenience, we make a choice of	*Common difference*
3 numbers in AP as $(a-d)$, a, $(a+d)$	d
4 numbers in AP as $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$	$2d$
5 numbers in AP as $(a-2d)$, $(a-d)$, a, $(a+d)$, $(a+2d)$	d
6 numbers in AP as $(a-5d)$, $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$, $(a+5d)$	$2d$

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1. The angles of a triangle are in AP.The greatest angle is twice the least. Find all the angles of triangle.

The sum of the first three terms of an AP is 33. If the product of the first and the third terms exceeds the second term by 29, find the AP.

Solution. Let the first three terms in an AP be : $(a-d)$, a, $(a+d)$

Sum of terms : $(a-d)+a+(a+d)=33 \Rightarrow 3a=33 \Rightarrow a=11$

Also, first term × third term = second term + 29

$\Rightarrow (a-d)(a+d)=a+29 \Rightarrow a^2-d^2=a+29$

$\Rightarrow 121-d^2=11+29 \Rightarrow d^2=81 \Rightarrow d=\pm 9$

Thus, $a=11$ and $d=\pm 9$

Hence, there are two APs given by 2, 11, 20,... and 20, 11, 2,...

2. Find the 12th term from the end of the AP: -2, -4, -6, ……--100

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Find the 12th term from the end of the AP : $-2, -4, -6, \ldots -100$

Solution. Here, $d = -4-(-2) = -2$, $l = -100$

12th term from the end of the AP

$$= l-(n-1)d$$

$$= -100-(12-1)\times(-2) = -100+22 = -78.$$

3.How many terms of the AP; -15,-13,-11,…….are needed to make the sum -55.Explain the reason for double answer.

How many terms of the AP : −15, −13, −11,... are needed to make the sum −55. Explain the reason for double answer.

Solution. Given the AP : −15, −13, −11,...

Here, $a = -15$, $d = -13 - (-15) = -13 + 15 = 2$

Let $S_n = -55$

Then, $\frac{n}{2}[2a + (n-1)d] = -55$

$\Rightarrow \frac{n}{2}[2 \times (-15) + (n-1) \times 2] = -55 \Rightarrow -15n + n(n-1) = -55$

$\Rightarrow n^2 - 16n + 55 = 0 \Rightarrow n^2 - 5n - 11n + 55 = 0$

$\Rightarrow n(n-5) - 11(n-5) = 0 \Rightarrow (n-5)(n-11) = 0$

$\Rightarrow n = 5$ or $n = 11$

Both values of n are admissible. Hence, the number of terms is either 5 or 11.

Reason for double answer

Here, the sum of first 5 terms = the sum of first 11 terms = −55. The two answers are possible because the sum of the terms from 6th to 11th will be zero. This is because a is negative and d is positive, so that some terms will be positive and some others negative, and will cancel out each other.

4. Solve the equation: -4 +(-1) + 2 +……+ x =437

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Solution. Here, the list of numbers : $-4, -1, 2, \ldots, x$ forms an AP with $a = -4$, $d = -1-(-4) = 3$, $a_n = x$

$$\Rightarrow \quad a_n = a + (n-1)d = x \quad \Rightarrow \quad -4 + (n-1)\times 3 = x \quad \Rightarrow \quad x = 3n - 7$$

Also, $$S = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad 437 = \frac{n}{2}[2\times(-4) + (n-1)\times 3]$$

$$\Rightarrow \quad 874 = n(3n-11) \quad \Rightarrow 3n^2 - 11n - 874 = 0$$

$$\therefore \quad n = \frac{11 \pm \sqrt{121 - 4\times 3\times(-874)}}{2\times 3}$$

$$= \frac{11 \pm \sqrt{121 + 10488}}{6} = \frac{11 \pm \sqrt{10609}}{6} = \frac{11 \pm 103}{6}$$

$$\Rightarrow \quad n = \frac{114}{6} \quad \text{or} \quad n = -\frac{46}{6}$$

$$\Rightarrow \quad n = 19 \quad \text{or} \quad n = -\frac{23}{3}$$

An n cannot be negative, so $n = 19$.

Hence, $x = 3n - 7 = 3\times 19 - 7 = 50$.

5. The sum of three numbers of an AP is 27 and their product is 405. Find the numbers.

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Let three numbers in AP are $a - d$, a and $a + d$

$\therefore \quad (a - d) + a + (a + d) = 27$

$\Rightarrow \quad 3a = 27$

$\Rightarrow \quad a = 9$

Also $\quad (a - d)\,(a)\,(a + d) = 405$

$\Rightarrow \quad (9 - d)\,(9)\,(9 + d) = 405$

$\Rightarrow \quad (9 - d)\,(9 + d) = 45$

$\Rightarrow \quad 81 - d^2 = 45$

$\Rightarrow \quad d^2 = 36$

$\Rightarrow \quad d = 6, -6$

When $d = 6$, numbers are 3, 9, 15

When $d = -6$, numbers are 15, 9, 3

HOME ASSIGNMENT Ex. 5.1(Q1 toQ16)

AHA

1. The first term of an AP is -5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.
2. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$.

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