

ARITHMETIC PROGRESSIONS

PPT-7

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 05
CHAPTER NAME : ARITHMETIC PROGRESSIONS

CHANGING YOUR TOMORROW

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PREVIOUS KNOWLEDGE TEST

- The n th term a_n of the AP with first term a and common difference d is given by $a_n = a + (n - 1)d$.
- a_n is also called the general term of the AP.
- n th term of an AP from the end = $l - (n - 1)d$
- Sum of the first n terms of AP; $S_n = \frac{n}{2}[2a + (n - 1)d]$
- OR
- $S_n = \frac{n}{2}(a + a_n)$
- SELECTION OF TERMS;
- For 3 numbers in AP $(a-d), a, (a+d)$
- For 4 numbers in AP $(a-3d), (a-d), (a+d), (a+3d)$

LEARNING OUTCOME

1. Students will be able to know AP as list of numbers in which successive term is obtained by adding a fixed number to previous term.
2. Students will be able to observe geometric patterns and understand the concept of AP
3. Students will be able to identify situations in daily life where the AP is observed.
4. Students will be able to identify the first term & common difference.
5. Students will be able to calculate the required term.
6. Students will be able to find n th term from the end of the AP.

- Short Tricks to solve AP.
- https://youtu.be/Op_yi9WXjBs

1. Find the sum of last ten terms of the AP: 8, 10, 12,, 126.

Solution. Given the AP : 8, 10, 12, ..., 126.

Here, $a=8$, $d=2$, $a_n=126$ (say)

$$\Rightarrow a+(n-1)d=126$$

$$\Rightarrow 8+(n-1)\times 2=126 \quad \Rightarrow (n-1)\times 2=118 \quad \Rightarrow n-1=59$$

$$\therefore n=60$$

Clearly, the last 10 terms of the AP will be from 51st term to 60th term, which also form an AP.

$$a_{51} = a + 50d = 8 + 50 \times 2 = 108$$

Also, $a_{60} = 126$

Hence, the sum of last ten terms of the given AP will be

$$S = \frac{10}{2} [a_{51} + a_{60}] = 5(108 + 126) = 5 \times 234 = 1170.$$

2.If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289,find the sum of first n terms

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Given, $S_7 = 49$

$$\Rightarrow 49 = \frac{7}{2}[2a + (7-1)d] \Rightarrow 7 \times 2 = [2a + 6d]$$

$$\Rightarrow 14 = 2a + 6d \Rightarrow a + 3d = 7 \quad \dots(1)$$

and $S_{17} = 289$

$$\Rightarrow 289 = \frac{17}{2}[2a + (17-1)d] \Rightarrow 2a + 16d = \frac{289 \times 2}{17} = 34$$

$$\Rightarrow a + 8d = 17 \quad \dots(2)$$

Now subtracting equation (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Putting the value of d in equation (1), we get

$$a + 3 \times 2 = 7 \Rightarrow a = 7 - 6 = 1$$

Here $a = 1$ and $d = 2$

Now,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = \frac{n}{2}[2 + 2n - 2] = \frac{n}{2} \times 2n = n^2$$

3. The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find the 28th term of this AP.

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Solution. Since, $S_n = \frac{n}{2} [2a + (n - 1)d]$

$\therefore S_7 = \frac{7}{2} [2a + (7 - 1)d] \Rightarrow S_7 = \frac{7}{2} [2a + 6d]$ [S₇ = 63 (given)]

$\Rightarrow 63 = 7a + 21d$

$\Rightarrow a = \frac{63 - 21d}{7}$... (1)

Also, $S_{14} = \frac{14}{2} [2a + 13d]$

$\Rightarrow S_{14} = 14a + 91d$

But according to question, $S_{1-7} + S_{8-14} = S_{14}$

$\Rightarrow 63 + 161 = 14a + 91d \Rightarrow 224 = 14a + 91d$

$\Rightarrow 2a + 13d = 32$

$\Rightarrow 126 - 42d + 91d = 224 \Rightarrow 2\left(\frac{63 - 21d}{7}\right) + 13d = 32$... (2)

$\Rightarrow 49d = 98 \Rightarrow d = 2$

$\therefore a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = \frac{21}{7} = 3$

Thus, $a_{28} = a + 27d = 3 + 27 \times 2$

$\Rightarrow a_{28} = 3 + 54 = 57$



4. If S_n denotes the sum of the first n terms of an AP, prove that

$$S_{30} = 3(S_{20} - S_{10})$$

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Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{30} = \frac{30}{2}[2a + (30-1)d] \Rightarrow S_{30} = \frac{30}{2}[2a + 29d]$

$\Rightarrow S_{30} = 15(2a + 29d) = 30a + 435d$... (1)

and $S_{20} = \frac{20}{2}[2a + (20-1)d] = \frac{20}{2}[2a + 19d]$

$$S_{20} = 10(2a + 19d) = 20a + 190d$$

$$S_{10} = \frac{10}{2}[2a + (10-1)d] \Rightarrow \frac{10}{2}[2a + 9d]$$

$\Rightarrow S_{10} = 5(2a + 9d) = 10a + 45d$

$$\begin{aligned} 3(S_{20} - S_{10}) &= 3[20a + 190d - 10a - 45d] \\ &= 3[10a + 145d] = 30a + 435d = S_{30} \end{aligned}$$

[From (1)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

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Given, $S_n = 3n^2 - 4n$... (1)

Replacing n by $n - 1$, we get

$$S_{n-1} = 3(n-1)^2 - 4(n-1) \quad \dots(2)$$

Since, $a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$
 $= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$
 $= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$

So, n th term, $a_n = 6n - 7$... (3)

Substituting $n = 1, 2, 3, \dots$ respectively in (3), we get

$$a_1 = 6 \times 1 - 7 = -1, a_2 = 6 \times 2 - 7 = 5$$

and $a_3 = 6 \times 3 - 7 = 11$

Hence, AP is $-1, 5, 11, \dots$

12th term, $a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$ [From (3)]

HOME ASSIGNMENT Ch -5

THANKING YOU
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