TRIANGLES
PPT-13

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 06
CHAPTER NAME :TRIANGLES

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

1 . If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other

2. Pythagoras Theorem ; : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

3.Converse of Pythagoras Theorem 6.9 : In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

4.The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ratio of areas of two similar triangles is equal to:

1.Ratio of the squares of their corresponding sides.

2. Ratio of the squares of their corresponding altitudes.

3. Ratio of the squares of their corresponding medians.

4. Ratio of the squares of their corresponding angle-bisector segments.

LEARNING OUTCOME

1. Students will be able to prove and apply Thales theorem (Basic Proportionality theorem.

2.Students will be able to know relation between the ratio of the areas of two similar triangles and the ratio of their corresponding sides.

3.Students will be able to prove: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

4.Students will be able to solve problems based on ratio of area of similar of triangles.

5.Students will be able to solve problems based on Pythagoras Theorem.

6.. Students will be able to solve problems based on converse of Pythagoras Theorem.

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1. BL and CM are medians of a triangle ABC right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$

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Solution. In right angled ΔBAL, we have

$$BL^2 = AL^2 + AB^2 \quad \text{[By Pythagoras Theorem]}$$

$$= \left(\frac{1}{2}AC\right)^2 + AB^2 \quad \left[\because AL = \frac{1}{2}AC\right]$$

$$\Rightarrow \quad 4BL^2 = AC^2 + 4AB^2 \quad \text{...(1)}$$

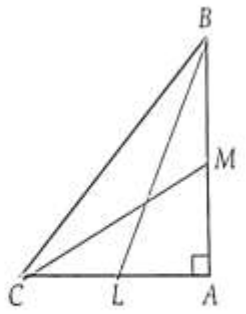

FIGURE 6.176

Also, in right angled ΔCAM, we have

$$CM^2 = AC^2 + AM^2 \quad \text{[By Pythagoras Theorem]}$$

$$CM^2 = AC^2 + \left(\frac{1}{2}AB\right)^2$$

$$\Rightarrow \quad 4CM^2 = 4AC^2 + AB^2 \quad \text{...(2)}$$

On adding (1) and (2), we get

$$4(BL^2 + CM^2) = 5AC^2 + 5AB^2 = 5(AC^2 + AB^2)$$

Hence, $4(BC^2 + CM^2) = 5BC^2 \quad [\because AC^2 + AB^2 = BC^2]$

3. O is any point inside a rectangle ABCD . Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Solution. Through O, draw $PQ \parallel AB$, so that P lies on AD and Q lies on BC.

$$PQ \parallel AB \quad \Rightarrow \quad PQ \perp BC \quad \text{and} \quad PQ \perp AD$$

$\Rightarrow$ Both $ABQP$ and $CDPQ$ are rectangles

$$\Rightarrow \quad AP = BQ \quad \text{and} \quad CQ = DP$$

[Opposite sides of rectangles]

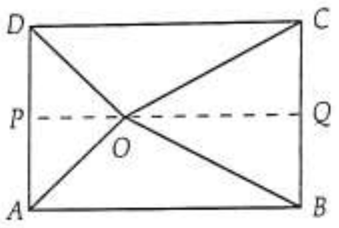

FIGURE 6.179

From right ΔOQB, $\quad OB^2 = OQ^2 + BQ^2$

From right ΔOPD, $\quad OD^2 = OP^2 + DP^2$

$$\therefore \quad OB^2 + OD^2 = OP^2 + OQ^2 + BQ^2 + DP^2 \quad ...(i)$$

From right ΔOPA, $\quad OA^2 = OP^2 + AP^2$

From right ΔOQC, $\quad OC^2 = OQ^2 + CQ^2$

$$\therefore \quad OA^2 + OC^2 = OP^2 + OQ^2 + AP^2 + CQ^2 = OP^2 + OQ^2 + BQ^2 + DP^2 \quad ...(ii)$$

$[AP = BQ, CQ = DP]$

From (i) and (ii), we get : $OB^2 + OD^2 = OA^2 + OC^2$.

4. Two isosceles triangles have equal vertical angles and their areas are in the ratio16:25.Find ratio of their corresponding heights

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Solution. In ΔABC and ΔDEF,

$$AB = AC \quad \text{and} \quad DE = DF$$

$$\Rightarrow \quad \frac{AB}{AC} = 1 = \frac{DE}{DF}$$

Also, $\angle A = \angle D$ [Given]

$\therefore \quad \Delta ABC \sim \Delta DEF$ [SAS similarity]

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AL^2}{DM^2}$$

[$AL \perp BC$ and $DM \perp EF$]

$$\Rightarrow \quad \frac{16}{25} = \frac{AL^2}{DM^2} \quad \Rightarrow \quad \frac{4}{5} = \frac{AL}{DM}.$$

Hence, $AL : DM = 4 : 5$.

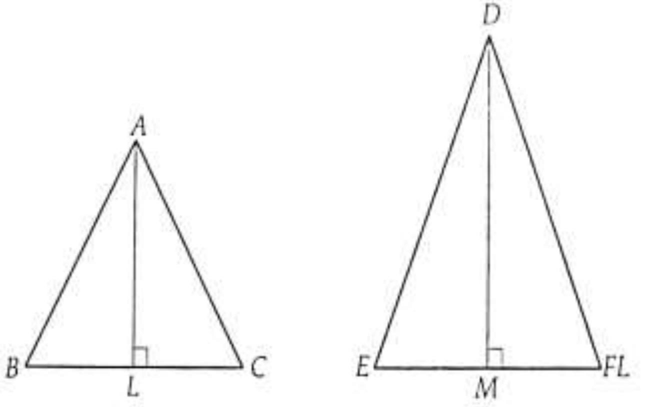

FIGURE 6.136

5.Prove that the ratio of perimeters of two similar triangles is same as the ratio of their corresponding sides

Solution. Let ΔABC and ΔDEF be two similar triangles. Then their corresponding sides will be proportional.

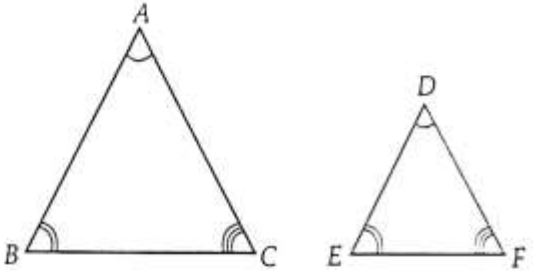

FIGURE 6.61

$$\therefore \quad \frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=k \quad \text{(say)} \qquad \text{...(1)}$$

$$\Rightarrow \quad AB=k.DE,\ \ BC=k.EF,\ \text{and}\ \ AC=k.DF$$

$$\therefore \quad \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}=\frac{AB+BC+AC}{DE+EF+DF}=\frac{k(DE+EF+DF)}{(DE+EF+DF)}=k \qquad \text{...(2)}$$

From (1) and (2), we get

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}=\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$$

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6.The perimeter of two similar triangles ABC & LMN are 60 cm and 48 cm respectively. If LM=8 cm, then what is the length of AB?

Solution. As the ratio of the perimeters of two similar Δs is same as the ratio of their orresponding sides,

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta LMN} = \frac{AB}{LM} \quad \Rightarrow \frac{60}{48} = \frac{AB}{8}$$

$$\therefore \quad AB = \frac{60}{48} \times 8 = 10 \text{ cm.}$$

6.The perimeter of two similar triangles ABC & LMN are 60 cm and 48 cm respectively. If LM=8 cm, then what is the length of AB?

7. Two poles of height p and q meters are standing vertically on a level ground, a meter apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is pq/(p+q).

Solution. In ΔCQP and ΔCAB,

$$\angle C = \angle C \qquad \text{[Common]}$$

$$\angle CQP = \angle CAB \qquad \text{[Each} = 90°\text{]}$$

$$\therefore \quad \Delta CQP \sim \Delta CAB \qquad \text{[AA similarity]}$$

$$\Rightarrow \quad \frac{CQ}{CA} = \frac{PQ}{AB} \quad \Rightarrow \quad \frac{x}{a} = \frac{h}{q} \qquad ...(1)$$

In ΔAQP and ΔACD,

$$\angle A = \angle A \qquad \text{[Common]}$$

$$\angle AQP = \angle ACD \qquad \text{[Each} = 90°\text{]}$$

$$\Rightarrow \quad \frac{AQ}{AC} = \frac{PQ}{CD} \quad \Rightarrow \quad \frac{a-x}{a} = \frac{h}{p} \qquad ...(2)$$

Adding (1) and (2),

$$\frac{x}{a} + \frac{a-x}{a} = \frac{h}{q} + \frac{h}{p} \quad \Rightarrow \quad \frac{x}{a} + 1 - \frac{x}{a} = h\left(\frac{1}{q} + \frac{1}{p}\right) \quad \Rightarrow \quad 1 = h\left(\frac{p+q}{pq}\right)$$

Hence, $h = \dfrac{pq}{p+q}$

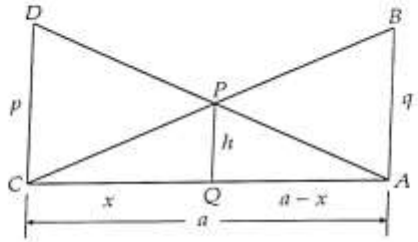

FIGURE 6.92

HOME ASSIGNMENT CH-6

THANKING YOU

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