TRIANGLES
PPT-8

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 06
CHAPTER NAME :TRIANGLES

CHANGING YOUR TOMORROW

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PREVIOUS KNOWLEDGE TEST

Two triangles are similar if either of the following three criterion's are satisfied:

1.AAA similarity Criterion. If two triangles are equiangular, then they are similar.

2.Corollary(AA similarity). If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

3.SSS Similarity Criterion. If the corresponding sides of two triangles are proportional, then they are similar.

3.SAS Similarity Criterion. If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

- **Results in Similar Triangles based on Similarity Criterion:**

1.Ratio of corresponding sides = Ratio of corresponding perimeters

2.Ratio of corresponding sides = Ratio of corresponding medians

3.Ratio of corresponding sides = Ratio of corresponding altitudes

4.Ratio of corresponding sides = Ratio of corresponding angle bisector segments

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LEARNING OUTCOME

1. .Students will be able to know relation between the ratio of the areas of two similar triangles and the ratio of their corresponding sides.

2.Students will be able to prove 'The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides'.

3. Students will be able to solve problems based on ratio of area of similar of triangles.

1. Theorem-6.6: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

https://youtu.be/210qR01kvEg (8.45)

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given. Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$.

To Prove. $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$

Construction. Draw $AL \perp BC$ and $DM \perp EF$.

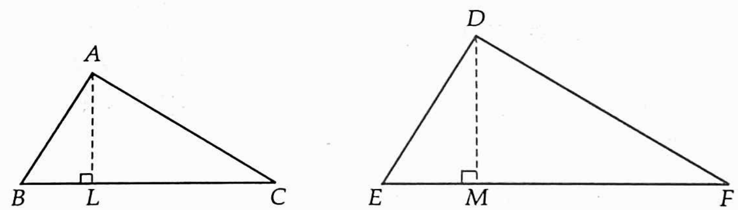

FIGURE 6.121

Proof. Since $\Delta ABC \sim \Delta DEF$, it follows that the triangles are equiangular and their sides are proportional.

$\therefore \quad \angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ...(*i*)

$\because$ Area of a $\Delta = \frac{1}{2}$ base $\times$ height

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given. Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$.

To Prove. $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2} = \dfrac{BC^2}{EF^2}$

Construction. Draw $AL \perp BC$ and $DM \perp EF$.

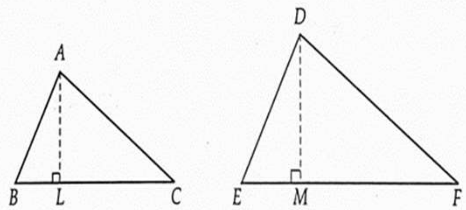

FIGURE 6.121

Proof. Since $\Delta ABC \sim \Delta DEF$, it follows that the triangles are equiangular and their sides are proportional.

$\therefore \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ ...(*i*)

$\because \quad$ Area of a $\Delta = \dfrac{1}{2}$ base $\times$ height

$\therefore \quad ar(\Delta ABC) = \dfrac{1}{2} BC \times AL$ and $ar(\Delta DEF) = \dfrac{1}{2} EF \times DM$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2} BC \times AL}{\frac{1}{2} EF \times DM} = \frac{BC}{EF} \times \frac{AL}{DM} \quad ...(ii)$$

Now, in ΔABL and ΔDEM, we have

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

$\angle ALB = \angle DME$ [Each $= 90°$]

$\therefore \quad \Delta ABL \sim \Delta DEM$ [*AA* similarity]

$\Rightarrow \quad \dfrac{AL}{DM} = \dfrac{AB}{DE}$ ...(*iii*)

Also, $\quad \dfrac{AB}{DE} = \dfrac{BC}{EF}$ [From (*i*)]

$\therefore \quad \dfrac{AL}{DM} = \dfrac{BC}{EF}$ ...(*iv*)

Using (*iv*) in (*ii*), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$$

Similarly, $\quad \dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2}$ and $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AC^2}{DF^2}$

Hence, $\quad \dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2} = \dfrac{BC^2}{EF^2}$.

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2. Let Δ ABC ~ Δ DEF and their areas be, respectively, 64 cm^2 and 121 cm^2 . If EF = 15.4 cm, find BC

Sol. Since, $\Delta ABC \sim \Delta DEF$

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

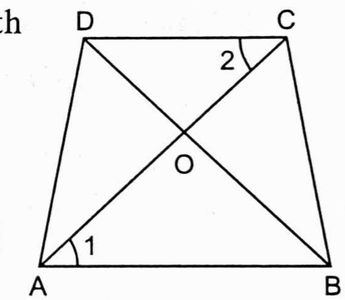
$$\Rightarrow \quad \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow \quad BC = \frac{8 \times 15.4}{11} = 11.2 \text{ cm}$$

3. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

3. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Sol. ABCD is a trapezium with AB || DC and AB = 2 CD



In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$ [V.O.A.]

$\angle 1 = \angle 2$ [Alternate angles]

$\therefore \Delta AOB \sim \Delta COD$

$$\therefore \frac{ar\,\Delta AOB}{ar\,\Delta COD} = \frac{AB^2}{CD^2}$$

$$= \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2} = \frac{4}{1}$$

HOME ASSIGNMENT Ex. 6.4 Q. No 1 to Q3

AHA

1.The line segment XY is parallel to side AC of Δ ABC and it divides the triangle into two parts of equal areas. Find the ratio AX /AB ·

THANKING YOU

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