ODM EDUCATIONAL GROUP
Changing your Tomorrow

TRIANGLES
PPT-9

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 06
CHAPTER NAME :TRIANGLES

CHANGING YOUR TOMORROW

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PREVIOUS KNOWLEDGE TEST

. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Results in Similar Triangles based on Similarity Criterion:

1.Ratio of corresponding sides = Ratio of corresponding perimeters

2.Ratio of corresponding sides = Ratio of corresponding medians

3.Ratio of corresponding sides = Ratio of corresponding altitudes

4.Ratio of corresponding sides = Ratio of corresponding angle bisector segments

LEARNING OUTCOME

1.Students will be able to know relation between the ratio of the areas of two similar triangles and the ratio of their corresponding sides.

2. Students will be able to solve problems based on ratio of area of similar of triangles

1. Theorem-6.6: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

https://youtu.be/210qR01kvEg (8.45)

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1.If the areas of two similar triangles are equal, prove that they are congruent.

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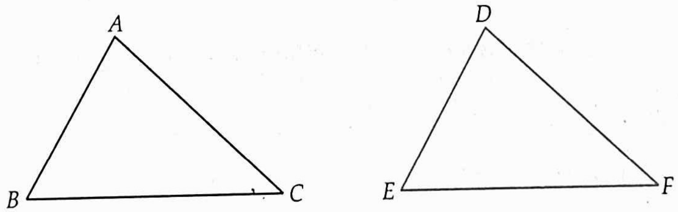
Solution. *Given.* $\Delta ABC \sim \Delta DEF$ such that ar $(\Delta ABC) =$ ar (ΔDEF).

To prove. $\Delta ABC \cong \Delta DEF$.



Proof. As the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides, so

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$

$\Rightarrow$ $\frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = 1$ $[\because \text{ar}(\Delta ABC) = \text{ar}(\Delta DEF)]$

$\Rightarrow$ $AB^2 = DE^2$, $AC^2 = DF^2$ and $BC^2 = EF^2$

$\Rightarrow$ $AB = DE$, $AC = DF$ and $BC = EF$

Hence, $\Delta ABC \cong \Delta DEF$. [$SSS$ congruency]

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2. E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

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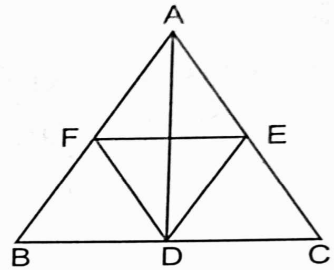
Sol. Given: D, E and F are mid-points of sides BC, CA and AB respectively.



Proof: D and E are mid-points of sides BC and CA respectively

$\therefore \quad DE = \frac{1}{2} AB$

[Line segment joining the mid-points of two sides of triangle is parallel to the third side and half of it.]

Similarly $\quad EF = \frac{1}{2} BC$ and $DF = \frac{1}{2} AC$

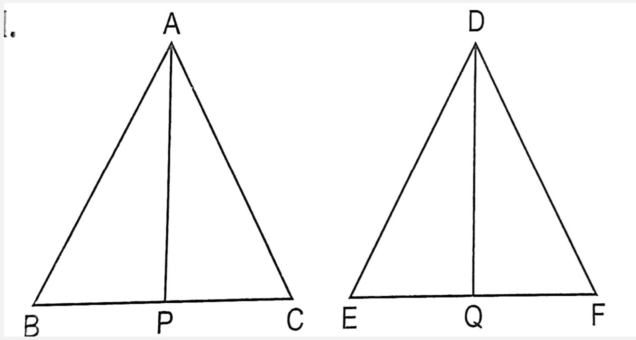
$\therefore \quad \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{1}{2}$

$\Rightarrow \quad \Delta DEF \sim \Delta ABC$ [SSS]

$\Rightarrow \quad \frac{\text{ar } \Delta DEF}{\text{ar } \Delta ABC} = \frac{DE^2}{AB^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Hence ar ΔDEF : ar $\Delta ABC = 1 : 4$

3.Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.



3.Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Given: $\Delta ABC \sim \Delta DEF$, AP and DQ are medians.

To Prove: $\dfrac{ar\ \Delta ABC}{ar\ \Delta DEF} = \dfrac{AP^2}{DQ^2}$

Proof: The ratio of the areas of two similar triangles is equal to the ratio of squares of two corresponding sides

$$\therefore \quad \frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AB^2}{DE^2}$$

$$\Delta ABC \sim \Delta DEF$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BP}{EQ} \quad \ldots(i)$$

$$\angle B = \angle E$$

[Corresponding angles of similar triangles]

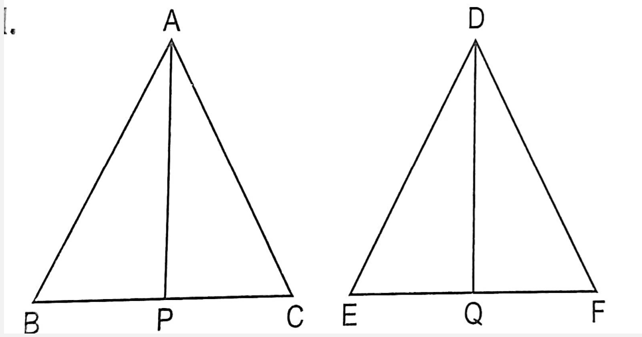
$$\therefore \quad \Delta ABP \sim \Delta DEQ \quad [SAS]$$

$$\therefore \quad \frac{BP}{EQ} = \frac{AP}{DQ} \quad \ldots(ii)$$

From equation (*i*) and (*ii*)

$$\therefore \quad \frac{AB}{DE} = \frac{AP}{DQ}$$

$$\therefore \quad \frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AP^2}{DQ^2}$$



4.Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

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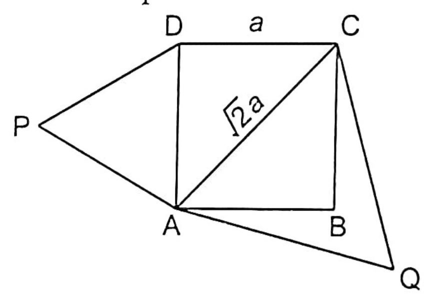
Sol. Let the side of the square ABCD be a



$$AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$$

$$AC = \sqrt{2}\,a$$

$\Delta PAD \sim \Delta QAC$ [AA similarity each angle = 60°]

$$\Rightarrow \quad \frac{ar\ \Delta PAD}{ar\ \Delta QAC} = \frac{AD^2}{AC^2} = \frac{a^2}{(\sqrt{2}\,a)^2} = \frac{1}{2}$$

$$\Rightarrow \quad ar\ \Delta PAD = \frac{1}{2}\ ar\ \Delta QAC$$

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(*a*) **2 : 3** (*b*) **4 : 9**

(*c*) **81 : 16** (*d*) **16 : 81**

Justification: Areas of two similar triangles are in the ratio of the squares of their corresponding sides.

$\therefore$ Ratio of areas of triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

or 16 : 81

Correct answer is (*d*)

HOME ASSIGNMENT Ex. 6.4 Q. No 4 to Q9

AHA

1. . D is a point on side BC of Δ ABC such that BD AB CD AC = · Prove that AD is the bisector of ∠ BAC.

THANKING YOU

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