

TRIANGLES

PPT-10

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 06
CHAPTER NAME : TRIANGLES

CHANGING YOUR TOMORROW

Website: www.odmegroup.org
Email: info@odmps.org

Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ratio of areas of two similar triangles is equal to:

1. Ratio of the squares of their corresponding sides.
2. Ratio of the squares of their corresponding altitudes.
3. Ratio of the squares of their corresponding medians.
4. Ratio of the squares of their corresponding angle-bisector segments.

LEARNING OUTCOME

1. Students will be able to know relation between hypotenuse and other two sides of a right angled triangle.
2. Students will be able to prove: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- 3.. Students will be able to solve problems based on Pythagoras theorem and apply in solving real life problems.

Theorem 6.7 : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other

. Pythagoras Theorem ; : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

<https://youtu.be/IV3qw8t4I6I> (10.05)

Theorem 6.7; If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Given. A triangle ABC in which
 $\angle BAC = 90^\circ$ and $AD \perp BC$

To Prove.

(i) $\triangle ADB \sim \triangle ABC$ (ii) $\triangle BDC \sim \triangle ABC$

(iii) $\triangle ADB \sim \triangle BDC$

Proof. (i) In $\triangle ADB$ and $\triangle ABC$, we have

$\angle ADB = \angle ABC$ [Each = 90°]

$\angle A = \angle A$ [Common]

$\therefore \triangle ADB \sim \triangle ABC$ [AA similarity]

(ii) In $\triangle BDC$ and $\triangle ABC$, we have

$\angle BDC = \angle ABC$ [Each = 90°]

$\angle C = \angle C$ [Common]

$\therefore \triangle BDC \sim \triangle ABC$ [AA similarity]

(iii) In $\triangle ABC$, $\angle A + \angle ABC + \angle C = 180^\circ$... (1)

In $\triangle ADB$, $\angle A + \angle ADB + \angle ABD = 180^\circ$... (2)

From (1) and (2), we get

$\angle A + \angle ABC + \angle C = \angle A + \angle ADB + \angle ABD$

$\Rightarrow \angle A + 90^\circ + \angle C = \angle A + 90^\circ + \angle ABD \Rightarrow \angle C = \angle ABD$

Now, in $\triangle ADB$ and $\triangle BDC$, we have

$\angle ABD = \angle C$ [Proved]

$\angle ADB = \angle BDC$ [Each = 90°]

$\therefore \triangle ADB \sim \triangle BDC$ [AA similarity]

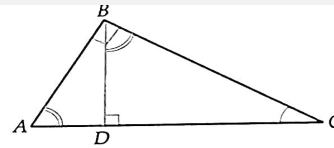


FIGURE 6.147

Theorem 6.8 : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides..

Given. A $\triangle ABC$ is which $\angle ABC = 90^\circ$.

To Prove. $AC^2 = AB^2 + BC^2$.

Construction. Draw $BD \perp AC$.

Proof. In $\triangle ADB$ and $\triangle ABC$, we have

$$\angle A = \angle A$$

[Common]

$$\angle ADB = \angle ABC$$

[Each = 90°]

$$\therefore \triangle ADB \sim \triangle ABC$$

[AA similarity]

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

[Corresponding sides are proportional]

$$\Rightarrow AD \times AC = AB^2 \quad \dots(1)$$

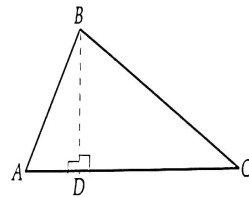


FIGURE 6.148

Now, in $\triangle BDC$ and $\triangle ABC$, we have

$$\angle C = \angle C$$

[Common]

$$\angle BDC = \angle ABC$$

[Each = 90°]

$$\therefore \triangle BDC \sim \triangle ABC$$

[AA similarity]

$$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$$

$$\Rightarrow DC \times AC = BC^2 \quad \dots(2)$$

On adding (1) and (2), we get

$$AD \times AC + DC \times AC = AB^2 + BC^2$$

$$\Rightarrow (AD + DC) \times AC = AB^2 + BC^2$$

$$\Rightarrow AC \times AC = AB^2 + BC^2$$

Hence, $AC^2 = AB^2 + BC^2$

REMARK Pythagoras Theorem was earlier stated by an ancient Indian Mathematician *Baudhayan* (about 800 B.C.) in the following form :

The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e., length and breadth).

For this reason, Pythagoras Theorem is also known as Baudhayan Theorem.

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 50 cm, 80 cm, 100 cm

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(i) 7 cm, 24 cm, 25 cm

$$\begin{aligned}(7)^2 + (24)^2 &= 49 + 576 \\ &= 625 = (25)^2 = 25\end{aligned}$$

∴ The given sides make a right angled triangle with hypotenuse 25 cm

) 50 cm, 80 cm, 100 cm

$$(100)^2 = 10000$$

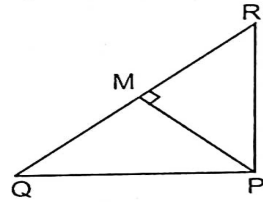
$$(80)^2 + (50)^2 = 6400 + 2500 = 8900$$

The square of larger side is not equal to the sum of squares of other two sides.

∴ The given triangle is not a right angled.

2. PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \cdot MR$.

Sol. In right angled ΔQPR ,
 $\angle P = 90^\circ$, $PM \perp QR$
 $\therefore \Delta PMQ \sim \Delta RMP$



[If \perp is drawn from the vertex of right angle to the hypotenuse then triangles on both sides of perpendicular are similar to each other, and to whole triangle]

$$\Rightarrow \frac{PM}{RM} = \frac{MQ}{MP}$$

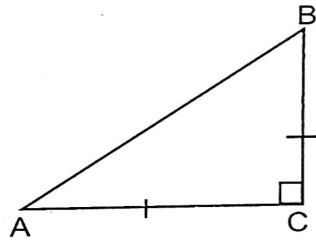
[Corresponding sides of similar triangles]

$$\Rightarrow PM \times MP = RM \times MQ \Rightarrow PM^2 = QM \cdot MR$$

3. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2 AC^2$.

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Sol. Given: In $\triangle ABC$, $\angle C = 90^\circ$ and $AC = BC$



To Prove: $AB^2 = 2AC^2$

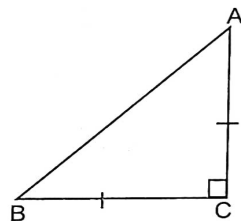
Proof: In $\triangle ABC$, $AB^2 = BC^2 + AC^2$

$$\begin{aligned} AB^2 &= AC^2 + AC^2 \text{ [Pythagoras theorem]} \\ &= 2AC^2 \end{aligned}$$

4. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2 AC^2$, prove that ABC is a right triangle.

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Sol. Given: In $\triangle ABC$, $AC = BC$ and $AB^2 = 2 AC^2$



To Prove: $\triangle ABC$ is a right angled triangle

Proof: $AB^2 = 2AC^2$
 $AB^2 = AC^2 + AC^2$
 $AB^2 = AC^2 + BC^2$ [$\because AC = BC$]
 $\Rightarrow \angle ACB = 90^\circ$
[By converse of Pythagoras theorem]
 $\therefore \triangle ABC$ is right angled triangle

HOME ASSIGNMENT Ex. 6.5 Q: No 1 to Q4

AHA

1. : O is any point inside a rectangle ABCD . Prove that $OB^2 + OD^2 = OA^2 + OC^2$

THANKING YOU
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