COORDINATE GEOMETRY
PPT-6

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 07
CHAPTER NAME : COORDINATE GEOMETRY

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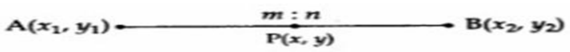
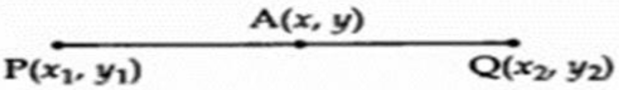
PREVIOUS KNOWLEDGE TEST

1. The coordinates of the point which divides the line segment joiningthe points A(x_1, y_1) and B(x_2, y_2) internally in the ratio m : n are:



$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

2.The mid-point of the line segment joining the points P (x_1, y_1) and Q ((x_2, y_2)



$$A(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

LEARNING OUTCOME

1.Students will be able to find the area of triangle when the coordinates of its vertices are given.

2. Students will be able to find the area of a quadrilateral when the coordinates of its vertices are given.

3. Students will be able to apply area of triangle to prove collinearity of three points .

4. Students will be able to apply area of triangle to find an unknown when three points are collinear.

Area of triangle;

https://youtu.be/hqU3pmMAbXM (9.59)

Algorithm for finding the area of a Δ when its vertices are given

Step 1 Write coordinates of the three vertices in three columns and repeat the coordinates of the first vertex.

Step 2 Draw the arrows pointing right downwards and pointing left downwards.

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

Step 3 We can calculate the area of the Δ as follows :

$$\text{area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\,|\,\text{Sum of the products pointing right downwards} - \text{Sum of the products pointing left downwards}\,|$$

$$= \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right|$$

. Collinearity Condition

If three points A, B and C are collinear and B lies between A and C, then,

- AB + BC = AC. AB, BC, and AC can be calculated using the distance formula.
- The ratio in which B divides AC, calculated using section formula for both the x and ycoordinates separately will be equal.
- Area of a triangle formed by three collinear points is zero.

1. . Find the area of the triangle whose vertices are : (i) (2, 3), (–1, 0), (2, – 4) (ii) (–5, –1), (3, –5), (5, 2

1. . Find the area of the triangle whose vertices are : (i) (2, 3), (–1, 0), (2, – 4) (ii) (–5, –1), (3, –5), (5, 2

(i) Here, $x_1 = 2, y_1 = 3, x_2 = -1, y_2 = 0,$
$x_3 = 2$ and $y_3 = -4$

$\therefore$ Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[\{2(0+4)\} + \{(-1)(-4-3)\} + \{2(3-0)\}]$$

$$= \frac{1}{2}[8 + 7 + 6] = \frac{21}{2} = \mathbf{10.5\ sq\ units.}$$

(ii) Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[\{-5(-5-2)\} + \{3(2+1)\} + \{5(-1+5)\}]$$

$$= \frac{1}{2}[35 + 9 + 20] = \frac{1}{2} \times 64 = \mathbf{32\ sq\ units.}$$

2. In each of the following find the value of ‘k’, for which the points are collinear. (i) (7, –2), (5, 1), (3, k) (ii) (8, 1), (k, – 4), (2, –5)

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(i) Points $A(\overset{x_1}{7}, \overset{y_1}{-2}), B(\overset{x_2}{5}, \overset{y_2}{1}), C(\overset{x_3}{3}, \overset{y_3}{k})$

For collinear points,

Area of $\Delta ABC = 0$

$$\text{Area of triangle ABC} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow \frac{1}{2}[7(1 - k) + 5(k + 2) + 3(-2 - 1)] = 0$$

$$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0 \Rightarrow 2k - 8 = 0 \Rightarrow 2k = 8$$

$$\Rightarrow k = \frac{8}{2} \Rightarrow k = 4$$

(ii) A(8, 1), B(k, –4), C(2, –5)

For collinear points,

Area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}[8(-4 + 5) + k(-5 - 1) + 2(1 + 4)] = 0$$

$$\Rightarrow 8 - 6k + 10 = 0 \Rightarrow 6k = 18 \Rightarrow k = 3$$

3. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3)..

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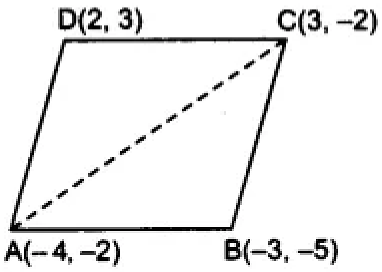

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-4 \times (-5 + 2) - 3(-2 + 2) + 3(-2 + 5)]$$

$$= \frac{1}{2}[12 + 0 + 9] = \frac{21}{2} \text{ square units}$$

$$\text{Area of } \Delta ADC = \frac{1}{2}[-4\,(-2 - 3) + 3\,(3 + 2) + 2 \times (-2 + 2)]$$

$$= \frac{1}{2}[20 + 15 + 0] = \frac{35}{2} \text{ square units}$$

$$\text{The area of quadrilateral ABCD} = \frac{21}{2} \text{ square units} + \frac{35}{2} \text{ square units}$$

$$= \frac{56}{2} \text{ square units} = 28 \text{ square units}$$

4. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

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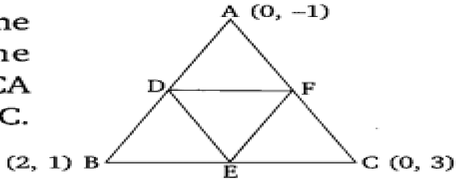
Let D, E and F be the mid-points of the sides AB, BC, and CA respectively of ΔABC. Then:



$$\text{Coordinates of D} = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$\text{Coordinates of E} = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$\text{Coordinates of F} = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

∴ Area of ΔDEF

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1 \times (2 - 1) + 1(1 - 0) + 0(0 - 2)]$$

$$= \frac{1}{2}[1 + 1 + 0] = \mathbf{1\ sq\ units}.$$

Area of ΔABC

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[0(1 - 3) + 2(3 + 1) + 0(-1 - 1)]$$

$$= \frac{1}{2}[0 + 8 + 0] = \frac{8}{2} = 4 \text{ sq units.}$$

∴ Ratio of area of ΔDEF to the area of ΔABC = **1 : 4**.

HOME ASSIGNMENT Ex. 7.3 Q. No 1 to Q5

AHA

1. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.
2. ABCD is a rectangle formed by the points A(–1, –1), B(– 1, 4), C(5, 4) and D(5, – 1). P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

THANKING YOU

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