INTRODUCTION TO TRIGONOMETRY

PPT-2

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

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PREVIOUS KNOWLEDGE TEST

- Trigonometric Ratios

Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Let ΔABC be a triangle right angled at B. Then the trigonometric ratios of the angle A in right ΔABC are defined as follows.

sine of $\angle A$ i.e. $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$

cosine of $\angle A$ i.e. $\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC}$

tangent of $\angle A$ i.e. $\tan A = \dfrac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \dfrac{BC}{AB}$

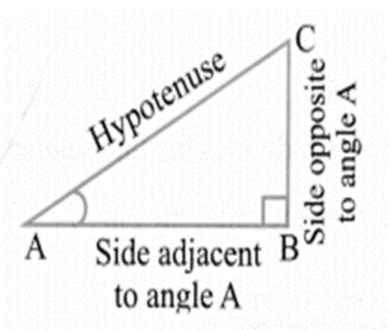
cosecant of $\angle A$ i.e. $\operatorname{cosec} A = \dfrac{1}{\text{sine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side opposite to } \angle A} = \dfrac{AC}{BC}$

secant of $\angle A$ i.e. $\sec A = \dfrac{1}{\text{cosine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side adjacent } \angle A} = \dfrac{AC}{AB}$

cotangent of $\angle A$ i.e. $\cot A = \dfrac{1}{\text{tangent of } \angle A} = \dfrac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \dfrac{AB}{BC}$

(i) $\sin A . \operatorname{cosec} A = 1$ (ii) $\cos A . \sec A = 1$

(iii) $\tan A . \cot A = 1$ (iv) $\tan A = \dfrac{\sin A}{\cos A}$ (v) $\cot A = \dfrac{\cos A}{\sin A}$



LEARNING OUTCOME

1 . Students will be able to know some ratios of the sides of a right triangle with respect to its acute angles.

2. Students will be able to know the relations between t- ratios.

3. Students will be able to apply and analyze trigonometry ratios in solving real life problems.

Trigonometric Ratios;

https://youtu.be/FTVJzHRRBfI (12.05)

1. Given 15 cot A = 8, find sin A and sec A

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$$15 \cot A = 8 \Rightarrow \cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let $AB = 8k$ and $BC = 15k$

In right angled ΔABC,

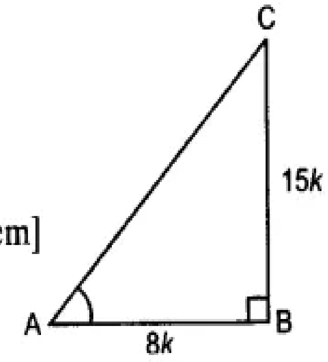
$$AC^2 = AB^2 + BC^2 \quad \text{[Pythagoras theorem]}$$

$$= (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2$$

$\Rightarrow$ $AC = \sqrt{289k^2} = 17k$

So, $\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

and $\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$



2. If cosec θ = 5/3, then what is the value of cos θ + tanθ

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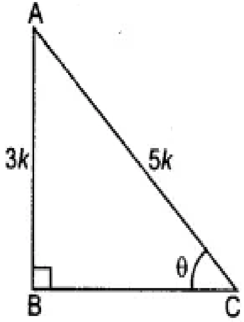
2. If cosec θ = 5/3, then what is the value of cos θ + tanθ

$\because \quad \operatorname{cosec}\theta = \frac{5}{3} \Rightarrow \frac{h}{p} = \frac{5}{3} \Rightarrow \frac{AC}{AB} = \frac{5}{3}$

In right angled ΔABC, $\angle B = 90°$, let $AC = 5K$, $AB = 3K$



So, $\quad AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow \quad (5k)^2 = (3k)^2 + BC^2$

$\Rightarrow \quad BC^2 = 25k^2 - 9k^2$

$\Rightarrow \quad BC^2 = 16k^2 \Rightarrow BC = 4k$

So, $\quad \cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$

Now, $\quad \cos\theta + \tan\theta = \frac{4}{5} + \frac{3}{4} = \frac{16+15}{20} = \frac{31}{20}$

3.If cot θ = 7 / 8 evaluate : (i) (1+ sin θ)(1- sin θ) /(1+ cos θ)(1 -cos θ) (ii) cot^2 θ

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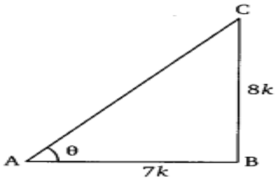
Solution:

$(1+\cos\theta)(1-\cos\theta)$

We have $\cot\theta = \frac{7}{8} = \frac{AB}{BC}$

Let $AB = 7k$ and $BC = 8k$.

Then in ΔABC,



$$AC^2 = AB^2 + BC^2$$
$$= (7k)^2 + (8k)^2 = 49k^2 + 64k^2 = 113k^2$$
$$\Rightarrow AC = k\sqrt{113}$$
$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$
and
$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}.$$

(i)
$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{\frac{49}{113}}{\frac{64}{113}} = \mathbf{\frac{49}{64}}.$$

(ii)
$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \mathbf{\frac{49}{64}}.$$
[From (i)]

4. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

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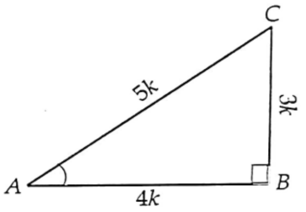
Solution. In right ΔABC, $\angle B = 90°$.

Given, $3 \cot A = 4 \Rightarrow \cot A = \frac{4}{3} = \frac{AB}{BC}$. If $AB = 4k$, then $BC = 3k$



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$$

$\therefore$ $AC = 5k$

Clearly, $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$, $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

and $\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

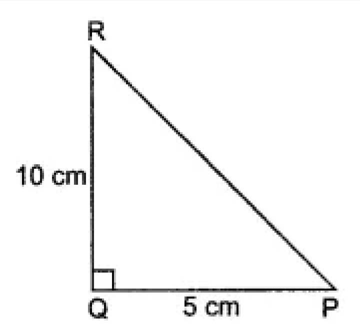
Now, $$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

Also, $$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$$

5. ΔRPQ is a right angled at Q. If PQ = 5 cm and RQ = 10 cm, find:

1. $\sin^2 P$
2. $\cos^2 R$ and $\tan R$
3. $\sin P \times \cos P$
4. $\sin^2 P - \cos^2 P$



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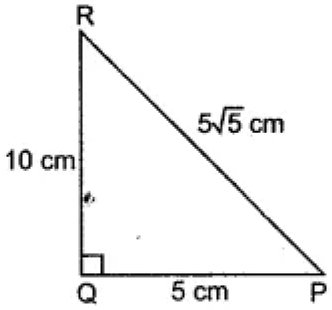
1. $\sin^2 P$
2. $\cos^2 R$ and $\tan R$
3. $\sin P \times \cos P$
4. $\sin^2 P - \cos^2 P$

In right angled ΔRPQ, $\angle Q = 90°$

So,
$$PR^2 = 10^2 + 5^2$$
$$PR^2 = 125$$
$$PR = 5\sqrt{5}\text{ cm}$$



(i) $$\sin^2 P = \left(\frac{10}{5\sqrt{5}}\right)^2 = \frac{4}{5}$$

(ii) $$\cos^2 R = \left(\frac{10}{5\sqrt{5}}\right)^2 = \frac{4}{5} \text{ and } \tan R = \frac{5}{10} = \frac{1}{2}$$

(iii) $$\sin P \times \cos P = \frac{10}{5\sqrt{5}} \times \frac{5}{5\sqrt{5}} = \frac{2}{5}$$

(iv) $$\sin^2 P - \cos^2 P = \left(\frac{10}{5\sqrt{5}}\right)^2 - \left(\frac{5}{5\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

HOME ASSIGNMENT Ex. 8.1 Q. No 4 to Q8

AHA

1. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm. Determine the values of sin Q and cos Q..

2. Consider Δ ACB, right-angled at C, in which AB = 29 units, BC = 21 units and ∠ ABC = θ Determine the values of (i) $\cos^2 \theta + \sin^2\theta$, (ii) $\sin^2\theta - \cos^2 \theta$.

THANKING YOU

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