INTRODUCTION TO TRIGONOMETRY

PPT-3

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

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PREVIOUS KNOWLEDGE TEST

- Trigonometric Ratios

Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Let ΔABC be a triangle right angled at B. Then the trigonometric ratios of the angle A in right ΔABC are defined as follows.

sine of $\angle A$ i.e. $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$

cosine of $\angle A$ i.e. $\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC}$

tangent of $\angle A$ i.e. $\tan A = \dfrac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \dfrac{BC}{AB}$

cosecant of $\angle A$ i.e. $\operatorname{cosec} A = \dfrac{1}{\text{sine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side opposite to } \angle A} = \dfrac{AC}{BC}$

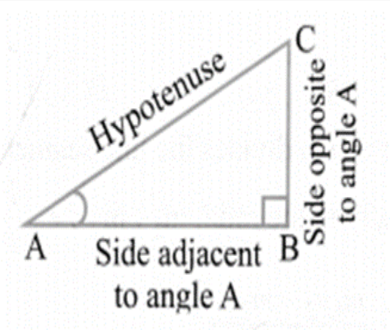
secant of $\angle A$ i.e. $\sec A = \dfrac{1}{\text{cosine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side adjacent } \angle A} = \dfrac{AC}{AB}$

cotangent of $\angle A$ i.e. $\cot A = \dfrac{1}{\text{tangent of } \angle A} = \dfrac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A}$

$= \dfrac{AB}{BC}$

(i) $\sin A . \operatorname{cosec} A = 1$ (ii) $\cos A . \sec A = 1$

(iii) $\tan A . \cot A = 1$ (iv) $\tan A = \dfrac{\sin A}{\cos A}$ (v) $\cot A = \dfrac{\cos A}{\sin A}$



LEARNING OUTCOME

1 . Students will be able to know some ratios of the sides of a right triangle with respect to its acute angles.

2. Students will be able to know the relations between t- ratios.

3. Students will be able to apply and analyze trigonometry ratios in solving real life problems.

Problem solving on Trigonometric Ratios;

https://youtu.be/63LC1ZHu8Vo (9.30)

1. In triangle ABC, right-angled at B, if tan A = 1/3 find the value of:

(i) sin A cos C + cos A sin C (ii) cos A cos C – sin A sin C

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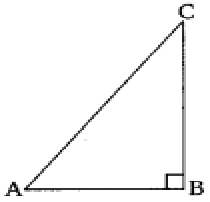
Let ABC is a right triangle at B.



$\because \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$\therefore \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let $AB = \sqrt{3}k$ and $BC = k$

Then by Pythagoras' Theorem, we have:

$$AC^2 = AB^2 + BC^2 = \left(\sqrt{3}k\right)^2 + (k)^2$$

$$\Rightarrow \quad AC = \sqrt{4k^2} = 2k \qquad \text{[Hypotenuse]}$$

Now $\sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \mathbf{1}.$$

(ii) cos A cos C – sin A sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \mathbf{0}.$$

2. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

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In right angled ΔPQR

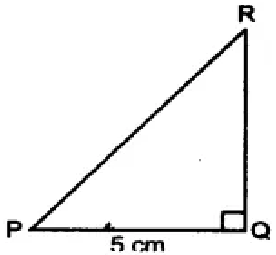
$$PR^2 = PQ^2 + QR^2 \Rightarrow PQ^2 = PR^2 - QR^2$$

$$\Rightarrow \quad (5)^2 = (PR + QR)(PR - QR)$$

$$\Rightarrow \quad 25 = 25(PR - QR) \Rightarrow \frac{25}{25} = PR - QR$$

$$\Rightarrow \quad PR - QR = 1 \quad \ldots (i)$$

$$\text{and} \quad PR + QR = 25 \quad \ldots (ii)$$



On adding equation (*i*) and (*ii*), we get

$$2PR = 26 \quad \Rightarrow \quad PR = \frac{26}{2} = 13 \text{ cm}$$

From equation (*i*),

$$PR - QR = 1 \quad \Rightarrow \quad QR = 13 - 1$$

$$QR = 12 \text{ cm}$$

$$\sin P = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

3.If $\sec\theta = \frac{5}{4}$, verify that $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$

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Solution. $\sec\theta = \frac{5}{4} = \frac{5k}{4k} = \frac{AC}{AB}$

By Pythagoras theorem, we have

$$BC^2 = AC^2 - AB^2$$

$$BC^2 = (5k)^2 - (4k)^2 = 25k^2 - 16k^2 = 9k^2$$

$$BC = 3k$$

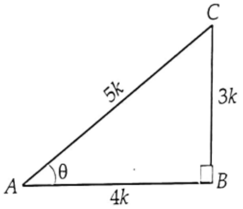

FIGURE 8.24

$\therefore \quad \sin\theta = \frac{BC}{AC} = \frac{3}{5}, \quad \tan\theta = \frac{BC}{AB} = \frac{3}{4}$

Now, $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{4}}{\frac{16+9}{16}} = \frac{3}{4}\times\frac{16}{25} = \frac{12}{25}$ and $\frac{\sin\theta}{\sec\theta} = \frac{3/5}{5/4} = \frac{3}{5}\times\frac{4}{5} = \frac{12}{25}$

Hence, $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$.

4. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

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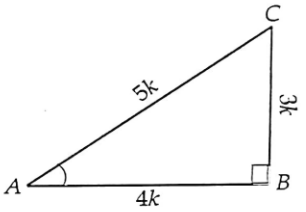
Solution. In right ΔABC, $\angle B = 90°$.

Given, $3\cot A = 4 \Rightarrow \cot A = \frac{4}{3} = \frac{AB}{BC}$. If $AB = 4k$, then $BC = 3k$



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$$

$\therefore$ $AC = 5k$

Clearly, $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$, $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

and $\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, $$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

Also, $$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$$

5. State whether the following statements are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = 5/2 for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = 4/3 for some angle.

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(*i*) It is true, because $\sin A = \frac{\text{side opposite to } \angle A \text{ (perpendicular)}}{\text{hyotenuse}}$ and in any right angle triangle, hypotenuse > perpendicular. So, sin A can never exceed 1. It may be 1 at the most when $\angle A = 90°$.

(*ii*) It is false, because perpendicular may be greater than its base in any right angled Δ and $\tan A = \frac{\text{perpendicular}}{\text{base}}$.

(*iii*) It is true, because $\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}}$ and the ratio of hypotenuse and adjacent side can be $\frac{5}{2}$.

(*iv*) It is false, because cos A is the abbreviation of cosine A.

(*v*) It is false, because it is not the product of cot and A but cotangent of angle A.

(*vi*) $\sin\theta = \frac{4}{3}$ is false, because $\sin\theta$ is the ratio of side opposite to angle θ and its hypotenuse. Opposite side is always less than the hypotenuse in right angled triangle. So $\sin\theta$ can never exceed unity.

HOME ASSIGNMENT Ex. 8.1 Q. No 9 to Q11

AHA

1. . If 5tan θ =4 find the value of 5sin θ -3cos θ /5sin θ +2 cos θ

2. . If ∠ B and ∠ Q are acute angles such that sin B = sin Q, then prove that ∠ B = ∠ Q.

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