

INTRODUCTION TO TRIGONOMETRY

PPT-5

SUBJECT : MATHEMATICS

CHAPTER NUMBER: 08

CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

CHANGING YOUR TOMORROW

PREVIOUS KNOWLEDGE TEST

● Trigonometric Ratios

Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Let ΔABC be a triangle right angled at B. Then the trigonometric ratios of the angle A in right ΔABC are defined as follows.

$$\text{sine of } \angle A \text{ i.e. } \sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\text{cosine of } \angle A \text{ i.e. } \cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A \text{ i.e. } \tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \frac{BC}{AB}$$

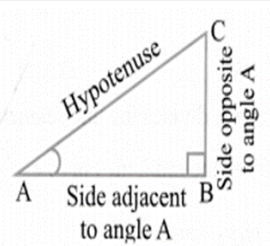
$$\text{cosecant of } \angle A \text{ i.e. } \text{cosec } A = \frac{1}{\text{sine of } \angle A} = \frac{\text{hypotenuse}}{\text{Side opposite to } \angle A} = \frac{AC}{BC}$$

$$\text{secant of } \angle A \text{ i.e. } \sec A = \frac{1}{\text{cosine of } \angle A} = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{AC}{AB}$$

$$\text{cotangent of } \angle A \text{ i.e. } \cot A = \frac{1}{\text{tangent of } \angle A} = \frac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \frac{AB}{BC}$$

(i) $\sin A \cdot \text{cosec } A = 1$ (ii) $\cos A \cdot \sec A = 1$

(iii) $\tan A \cdot \cot A = 1$ (iv) $\tan A = \frac{\sin A}{\cos A}$ (v) $\cot A = \frac{\cos A}{\sin A}$



The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains same.

Trigonometric Ratios of Some Specific Angles

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\text{cosec } A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

LEARNING OUTCOME

- 1 . Students will be able to know the trigonometric ratios of some specific angles.
2. Students will be able to know the relations between t- ratios.
3. Students will be able to apply and analyze trigonometric ratios of some specific angles in solving real life problems.

Problem solving on Trigonometric Ratio of
some specific angles;

<https://youtu.be/qv9e4z3wP5E> (9.50)

1. If $\sin (A + B) = 1$ and $\tan (A - B) = 1/\sqrt{3}$, find the value of;
(I) $\tan A + \cot B$
(II) $\sec A - \operatorname{cosec} B$

1. If $\sin (A + B) = 1$ and $\tan (A - B) = 1/\sqrt{3}$, find the value of;
 (I) $\tan A + \cot B$
 (II) $\sec A - \operatorname{cosec} B$

$$\begin{aligned} & \sin (A + B) = 1 && \text{(Given)} \\ \Rightarrow & \sin (A + B) = \sin 90^\circ && \text{(As } \sin 90^\circ = 1) \\ \Rightarrow & A + B = 90^\circ && \dots(i) \\ \text{Also} & \tan (A - B) = \frac{1}{\sqrt{3}} && \text{(Given)} \\ & \tan (A - B) = \tan 30^\circ && \text{(As } \tan 30^\circ = \frac{1}{\sqrt{3}}) \\ \Rightarrow & A - B = 30^\circ && \dots(ii) \end{aligned}$$

Solving (i) and (ii) for A and B, we get $A = 60^\circ$ and $B = 30^\circ$

$$(i) \tan A + \cot B = \tan 60^\circ + \cot 30^\circ = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$$

$$(ii) \sec A - \operatorname{cosec} B = \sec 60^\circ - \operatorname{cosec} 30^\circ = 2 - 2 = 0$$

2 .If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = 1 / \sqrt{3}$; $0^\circ < A + B \leq 90^\circ$; $A > B$,
find A and B.

2. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = 1 / \sqrt{3}$; $0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.

$$\because \tan(A - B) = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad A - B = 30^\circ \quad \dots(i)$$

$$\text{and } \tan(A + B) = \sqrt{3} \quad \Rightarrow \quad A + B = 60^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\begin{array}{r} A - B = 30^\circ \\ A + B = 60^\circ \\ \hline 2A = 90^\circ \Rightarrow A = 45^\circ \end{array}$$

Putting $A = 45^\circ$ in (i), we get

$$\begin{array}{r} 45^\circ - B = 30^\circ \\ B = 15^\circ \end{array}$$

\Rightarrow

Hence, $A = 45^\circ$ and $B = 15^\circ$.

3. Given that $\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$, find the value of $\cos 15^\circ$ in two ways. 1. Taking $A = 60^\circ$, $B = 45^\circ$ and 2. Taking $A = 45^\circ$, $B = 30^\circ$

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(i) By taking $A = 60^\circ$ and $B = 45^\circ$

$$\begin{aligned}\cos 15^\circ &= \cos(60^\circ - 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1 + \sqrt{3}}{2\sqrt{2}}\end{aligned}$$

(ii) By taking $A = 45^\circ$ and $B = 30^\circ$

$$\begin{aligned}\cos 15^\circ &= \cos(45^\circ - 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}\end{aligned}$$

4. If $\sin A = \cos A$, find the value of $2\tan^2 A + \sin^2 A - 1$.

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$$\begin{aligned} \because \quad & \sin A = \cos A \\ \Rightarrow \quad & A = 45^\circ \\ \text{Now,} \quad & 2\tan^2 A + \sin^2 A - 1 = 2\tan^2 45^\circ + \sin^2 45^\circ - 1 \\ & = 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 + \frac{1}{2} - 1 = \frac{3}{2} \end{aligned}$$

5. ABC is a triangle right angled at C and $AC = \sqrt{3} BC$. Prove that $\angle ABC = 60^\circ$

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Here, in $\triangle ABC$, $\angle C = 90^\circ$ and $AC = \sqrt{3} BC$

$$\Rightarrow \frac{AC}{BC} = \sqrt{3}$$

Also, $\tan B = \frac{AC}{BC}$

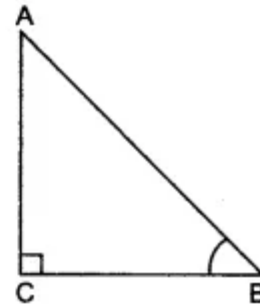
$$\Rightarrow \tan B = \sqrt{3}$$

$$\Rightarrow B = 60^\circ$$

[given]

...(i)

[Using (i)]



HOME ASSIGNMENT Ex. 8.2 Q: No 2 to 4

AHA

1. A rhombus of side 10cm has two angles 60° each. Find the lengths of diagonals of the rhombus..

THANKING YOU
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