INTRODUCTION TO TRIGONOMETRY

PPT-6

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

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PREVIOUS KNOWLEDGE TEST

- Trigonometric Ratios

Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Let ΔABC be a triangle right angled at B. Then the trigonometric ratios of the angle A in right ΔABC are defined as follows.

sine of ∠A i.e. $\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$

cosine of ∠A i.e. $\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$

tangent of ∠A i.e. $\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \frac{BC}{AB}$

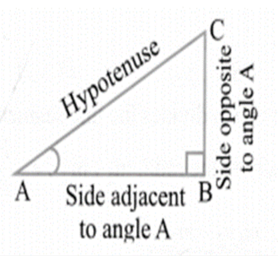
cosecant of ∠A i.e. $\operatorname{cosec} A = \frac{1}{\text{sine of } \angle A} = \frac{\text{hypotenuse}}{\text{Side opposite to } \angle A} = \frac{AC}{BC}$

secant of ∠A i.e. $\sec A = \frac{1}{\text{cosine of } \angle A} = \frac{\text{hypotenuse}}{\text{Side adjacent } \angle A} = \frac{AC}{AB}$

cotangent of ∠A i.e. $\cot A = \frac{1}{\text{tangent of } \angle A} = \frac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \frac{AB}{BC}$

(i) $\sin A . \operatorname{cosec} A = 1$ (ii) $\cos A . \sec A = 1$

(iii) $\tan A . \cot A = 1$ (iv) $\tan A = \frac{\sin A}{\cos A}$ (v) $\cot A = \frac{\cos A}{\sin A}$



The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains same.

Trigonometric Ratios of Some Specific Angles

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

LEARNING OUTCOME

1 . Students will be able to know the trigonometric ratios of complementary angles.

2.Students will be able to know the relations between t- ratios.

3. Students will be able to apply and analyze trigonometric ratios of complementary angles in solving real life problems.

4. Students will be able to solve problems involving trigonometric ratios of complementary angles .

Trigonometric Ratio of complementary angles

; https://youtu.be/AXVB-jpStXM (10.5)

Trigonometric ratios of complementary angles

Complementary angles. *Two angles are said to be complementary if their sum equals 90°.*

Trigonometric ratios of complementary angles. Consider right ΔABC, right-angled at B. Clearly, $\angle A$ and $\angle C$ form a complementary pair because

$$\angle A + \angle C = 90°$$

If $\angle A = \theta$, then $\angle C = 90° - \theta$

From right ΔABC, we have

$$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}$$

$$\text{cosec}\,\theta = \frac{AC}{BC} \qquad \sec\theta = \frac{AC}{AB} \qquad \cot\theta = \frac{AB}{BC}$$

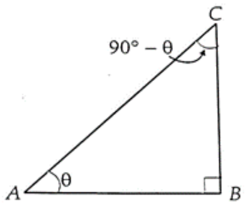


FIGURE 8.50

We now write t-ratios of complementary $\angle C = 90° - \theta$.

$$\sin(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{hypoenuse}} = \frac{BC}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{side adjacent to } \angle C} = \frac{AB}{BC} = \cot\theta$$

$$\text{cosec}(90° - \theta) = \frac{\text{hypotenuse}}{\text{side opposite to } \angle C} = \frac{AC}{AB} = \sec\theta$$

$$\sec(90° - \theta) = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle C} = \frac{AC}{BC} = \text{cosec}\,\theta$$

$$\cot(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{side opposite to } \angle C} = \frac{BC}{AB} = \tan\theta$$

1. Evaluate: (i) $\sin 18° / \cos 72°$

(ii) $\tan 26° / \cot 64°$

(iii) $\cos 48° - \sin 42°$

(iv) $\operatorname{cosec} 31° - \sec 59°$

1. Evaluate: (i) sin 18° /cos 72 °
(ii) tan 26°/ cot 64 °
(iii) cos 48° – sin 42°
(iv) cosec 31° – sec 59°

(i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = \mathbf{1}$

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = \mathbf{1}$

(iii) $\cos 48° - \sin 42° = \cos(90° - 42°) - \sin 42°$

$= \sin 42° - \sin 42° = \mathbf{0}$

(iv) $\text{cosec } 31° - \sec 59°$

$= \text{cosec } 31° - \sec(90° - 31°)$

$= \text{cosec } 31° - \text{cosec } 31° = \mathbf{0}$

2.Evaluate tan 65°/ cot 25° .

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We know : cot A = tan (90° – A)

So, cot 25° = tan (90° – 25°) = tan 65°

i.e., tan 65°/ cot 25° = tan 65°/ tan 65° = 1

3. If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

3. If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

We are given that sin 3A = cos (A – 26°).

Since sin 3A = cos (90° – 3A),

we can write ; cos (90° – 3A) = cos (A – 26°) (Since 90° – 3A and A – 26° are both acute angles),

therefore, 90° – 3A = A – 26°

which gives A = 29°

4.Show that:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ = \sqrt{3}$

(iii) $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots\ldots\ldots \tan 89^\circ = 1$

4.Show that:

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

1. (i) $\tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \tan 23° \tan(90° - 48°) \tan(90° - 23°)$

$= \tan 48° \tan 23° \cot 48° \cot 23°$

$= \tan 48° \tan 23° . \dfrac{1}{\tan 48°} . \dfrac{1}{\tan 23°}$

$= 1.$

4.Show that:

(ii) tan7° tan23° tan60° tan67° tan 83° =√3

(ii) $\tan 7° \tan 23° \tan 60° \tan 67° \tan 83°$

$= \tan 7° . \tan 83° . \tan 23° \tan 67° . \tan 60°$

$= \tan 7° . \tan(90° - 7°) . \tan 23° \tan(90° - 23°) . \tan 60°$

$= (\tan 7° . \cot 7°) . (\tan 23° . \cot 23°) . \tan 60°$

$= 1.1.\sqrt{3}$

$= \sqrt{3}.$

4.Show that:

(iii) $\tan 1° \tan 2° \tan 3° \ldots\ldots\ldots\ldots\ldots \tan 89° = 1$

$\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 89°$

$= \tan 1° \tan 2° \ldots \tan 44° \tan 45° \tan 46° \ldots \tan 88° \tan 89°$

$= [\tan 1° . \tan 89°].[\tan 2° . \tan 88°] \ldots [\tan 44° . \tan 46°]. \tan 45°$

$= [\tan 1° . \tan(90° - 1°)].[\tan 2° . \tan(90° - 2°)] \ldots [\tan 44° . \tan(90° - 44°)]. \tan 45°$

$= [\tan 1° . \cot 1°].[\tan 2° . \cot 2°] \ldots . [\tan 44° . \cot 44°]. \tan 45°$

$[\tan\theta . \cot\theta = 1,\ \tan 45° = 1]$

$= 1.1 \ldots\ldots 1.1 = 1.$

5. . If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

∴

5. . If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

We have $\tan 2A = \cot (A - 18°)$

$\Rightarrow \cot (90° - 2A) = \cot (A - 18°)$ [$\because \cot (90° - \theta) = \tan \theta$]

$\Rightarrow 90° - 2A = A - 18°$

$\Rightarrow 2A + A = 90° + 18°$

$\Rightarrow 3A = 108°$

$\Rightarrow A = \frac{108}{3} = 36°$.

HOME ASSIGNMENT Ex. 8.3 Q. No 1 to 3

AHA

1. Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

2. If A, B and C are interior angles of a triangle ABC, then prove that $\tan\left(\frac{A+B}{2}\right)=\cot\left(\frac{C}{2}\right)$

THANKING YOU

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