ODM EDUCATIONAL GROUP
Changing your Tomorrow

INTRODUCTION TO TRIGONOMETRY

PPT-7

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

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PREVIOUS KNOWLEDGE TEST

- Trigonometric Ratios

Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Let ΔABC be a triangle right angled at B. Then the trigonometric ratios of the angle A in right ΔABC are defined as follows.

sine of ∠A i.e. $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$

cosine of ∠A i.e. $\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC}$

tangent of ∠A i.e. $\tan A = \dfrac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \dfrac{BC}{AB}$

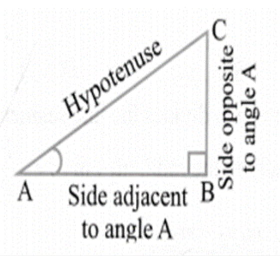
cosecant of ∠A i.e. $\text{cosec } A = \dfrac{1}{\text{sine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side opposite to } \angle A} = \dfrac{AC}{BC}$

secant of ∠A i.e. $\sec A = \dfrac{1}{\text{cosine of } \angle A} = \dfrac{\text{hypotenuse}}{\text{Side adjacent } \angle A} = \dfrac{AC}{AB}$

cotangent of ∠A i.e. $\cot A = \dfrac{1}{\text{tangent of } \angle A} = \dfrac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \dfrac{AB}{BC}$

(i) $\sin A . \text{cosec } A = 1$ (ii) $\cos A . \sec A = 1$

(iii) $\tan A . \cot A = 1$ (iv) $\tan A = \dfrac{\sin A}{\cos A}$ (v) $\cot A = \dfrac{\cos A}{\sin A}$



The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains same.

Trigonometric Ratios of Some Specific Angles

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

LEARNING OUTCOME

1 . Students will be able to know the trigonometric ratios of complementary angles.

2.Students will be able to know the relations between t- ratios.

3. Students will be able to apply and analyze trigonometric ratios of complementary angles in solving real life problems.

4. Students will be able to solve problems involving trigonometric ratios of complementary angles .

Problem solving on Trigonometric Ratio of complementary angles; https://youtu.be/lAmXPXvVrS0 (8.15).

Trigonometric ratios of complementary angles

Complementary angles. *Two angles are said to be complementary if their sum equals 90°.*

Trigonometric ratios of complementary angles. Consider right ΔABC, right-angled at B. Clearly, $\angle A$ and $\angle C$ form a complementary pair because

$$\angle A + \angle C = 90°$$

If $\angle A = \theta$, then $\angle C = 90° - \theta$

From right ΔABC, we have

$$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} \qquad \sec\theta = \frac{AC}{AB} \qquad \cot\theta = \frac{AB}{BC}$$

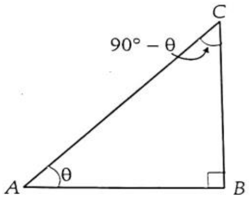


FIGURE 8.50

We now write *t*-ratios of complementary $\angle C = 90° - \theta$.

$$\sin(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{hypoenuse}} = \frac{BC}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{side adjacent to } \angle C} = \frac{AB}{BC} = \cot\theta$$

$$\operatorname{cosec}(90° - \theta) = \frac{\text{hypotenuse}}{\text{side opposite to } \angle C} = \frac{AC}{AB} = \sec\theta$$

$$\sec(90° - \theta) = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle C} = \frac{AC}{BC} = \operatorname{cosec}\theta$$

$$\cot(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{side opposite to } \angle C} = \frac{BC}{AB} = \tan\theta$$

1. If tan A = cot B, prove that A + B = 90°.

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$\tan A = \cot B$

$\Rightarrow \tan A = \tan(90° - B)$ [$\because \tan(90° - \theta) = \cot\theta$]

$\Rightarrow A = 90° - B$

$\Rightarrow A + B = 90°$

Proved.

2.If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A

2.If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A

We have $\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$ $[\because \operatorname{cosec}(90^\circ - \theta) = \sec\theta]$

$\Rightarrow 90^\circ - 4A = A - 20^\circ \Rightarrow 90^\circ + 20^\circ = 5A$

$\Rightarrow 110^\circ = 5A \Rightarrow \frac{110}{5} = A$

$\Rightarrow A = 22^\circ$

3.If $\sin\theta+\cos\theta=\sqrt{2}\ \sin(90-\theta)$,find $\cot\theta$

3.If sinθ+cosθ=√2 sin(90-θ),find cotθ

$$\sin\theta+\cos\theta=\sqrt{2}\sin(90^\circ-\theta)$$

$$\Rightarrow \quad \sin\theta+\cos\theta=\sqrt{2}\cos\theta$$

Dividing both sides by $\sin\theta$, we get

$$\frac{\sin\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}=\sqrt{2}\,\frac{\cos\theta}{\sin\theta}$$

$$1+\cot\theta=\sqrt{2}\cot\theta \quad \text{or} \quad 1=(\sqrt{2}-1)\cot\theta$$

Hence,
$$\cot\theta=\frac{1}{\sqrt{2}-1}=\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{(\sqrt{2}+1)}{2-1}=\sqrt{2}+1.$$

4. If A, B and C are interior angles of a triangle ABC, then show that (i) $sin\frac{B+C}{2} = cos\frac{A}{2}$ (ii) $tan\frac{A+B}{2} = cot\frac{C}{2}$

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4. If A, B and C are interior angles of a triangle ABC, then show that (i) $sin\frac{B+C}{2} = cos\frac{A}{2}$ (ii) $tan\frac{A+B}{2} = cot\frac{C}{2}$

The sum of the angles of a triangle is 180°.

$\therefore \quad A + B + C = 180°$

$\Rightarrow \quad B + C = 180° - A$

$\Rightarrow \quad \frac{B+C}{2} = 90° - \frac{A}{2}$ [Dividing both sides by 2]

$\Rightarrow \quad \sin\frac{B+C}{2} = \sin\left(90° - \frac{A}{2}\right)$

Hence, $\sin\frac{B+C}{2} = \cos\frac{A}{2}$. [$\because \sin(90° - \theta) = \cos\theta$]

4. If A, B and C are interior angles of a triangle ABC, then show that (i) $sin\frac{B+C}{2} = cos\frac{A}{2}$ (ii) $tan\frac{A+B}{2} = cot\frac{C}{2}$

$$A + B + C = 180° \quad [\angle s \text{ of a } \Delta]$$

$$\Rightarrow \quad \frac{A+B}{2} = \frac{180° - C}{2}$$

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right) = \tan\left(90° - \frac{C}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}. \quad [\because \tan(90° - \theta) = \cot\theta]$$

5. . If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

5. . If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

We have $\tan 2A = \cot (A - 18^\circ)$

$\Rightarrow \cot (90^\circ - 2A) = \cot (A - 18^\circ)$ [$\because \cot (90^\circ - \theta) = \tan \theta$]

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow 2A + A = 90^\circ + 18^\circ$

$\Rightarrow 3A = 108^\circ$

$\Rightarrow A = \frac{108}{3} = 36^\circ$.

HOME ASSIGNMENT Ex. 8.3 Q. No 4 to 7

AHA

1. .Evaluate; $cosec(65+\theta) - sec(25-\theta) - tan(55-0) + cot(35^0+\theta)$

THANKING YOU

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