INTRODUCTION TO TRIGONOMETRY

PPT-8

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

CHANGING YOUR TOMORROW

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

Complementary angles. *Two angles are said to be complementary if their sum equals 90°.*

Trigonometric ratios of complementary angles. Consider right ΔABC, right-angled at B. Clearly, $\angle A$ and $\angle C$ form a complementary pair because

$$\angle A + \angle C = 90°$$

If $\angle A = \theta$, then $\angle C = 90° - \theta$

From right $\Delta\, ABC$, we have

$$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} \qquad \sec\theta = \frac{AC}{AB} \qquad \cot\theta = \frac{AB}{BC}$$

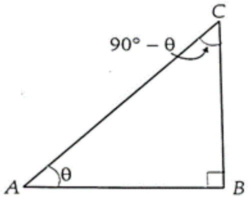


FIGURE 8.50

We now write *t*-ratios of complementary $\angle C = 90° - \theta$.

$$\sin(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{hypoenuse}} = \frac{BC}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{\text{side opposite to } \angle C}{\text{side adjacent to } \angle C} = \frac{AB}{BC} = \cot\theta$$

$$\operatorname{cosec}(90° - \theta) = \frac{\text{hypotenuse}}{\text{side opposite to } \angle C} = \frac{AC}{AB} = \sec\theta$$

$$\sec(90° - \theta) = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle C} = \frac{AC}{BC} = \operatorname{cosec}\theta$$

$$\cot(90° - \theta) = \frac{\text{side adjacent to } \angle C}{\text{side opposite to } \angle C} = \frac{BC}{AB} = \tan\theta$$

LEARNING OUTCOME

1 . Students will be able to know the Trigonometric Identities.

2. Students will be able to solve the problems involving Trigonometric Identities

3. Students will be able to apply and analyze Trigonometric Identities in solving problems.

4. Students will be able to convert t- ratios in terms of other t-ratios..

Trigonometric Identities :
https://youtu.be/tsn0yTgzNBE (11.45)

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = \cos\theta/\sin\theta$

1. $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

2. $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \mathrm{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \cot^2\theta = \mathrm{cosec}^2\theta - 1$

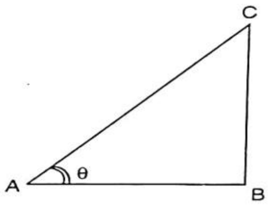
3. $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

Proof. In right angled triangle



(i) $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$$\sin\theta = \frac{BC}{AC} \Rightarrow \sin^2\theta = \frac{BC^2}{AC^2} \qquad \cos\theta = \frac{AB}{AC} \Rightarrow \cos^2\theta = \frac{AB^2}{AC^2}$$

On adding $\sin^2\theta + \cos^2\theta = \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2}$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta = \frac{BC^2 + AB^2}{AC^2} = \frac{AC^2}{AC^2} = 1$ [By Pythagoras Theorem, $AC^2 = BC^2 + AB^2$]

$\Rightarrow$ $\sin^2\theta + \cos^2\theta = 1$ **Proved.**

(ii) $\boxed{1 + \tan^2\theta = \sec^2\theta}$

$$\sec\theta = \frac{AC}{AB} \Rightarrow \sec^2\theta = \frac{AC^2}{AB^2} \qquad \tan\theta = \frac{BC}{AB} \Rightarrow \tan^2\theta = \frac{BC^2}{AB^2}$$

$\Rightarrow$ $1 + \tan^2\theta = 1 + \frac{BC^2}{AB^2} = \frac{AB^2 + BC^2}{AB^2}$ ($AB^2 + BC^2 = AC^2$, By Pythagoras Theorem)

$\Rightarrow$ $1 + \tan^2\theta = \sec^2\theta$ **Proved.**

(iii) $\boxed{1 + \cot^2\theta = \text{cosec}^2\theta}$

$$\text{cosec}\,\theta = \frac{AC}{BC} \Rightarrow \text{cosec}^2\theta = \frac{AC^2}{BC^2} \qquad \cot\theta = \frac{AB}{BC} \Rightarrow \cot^2\theta = \frac{AB^2}{BC^2}$$

$\Rightarrow$ $1 + \cot^2\theta = 1 + \frac{AB^2}{BC^2} = \frac{BC^2 + AB^2}{BC^2}$

$= \frac{AC^2}{BC^2}$ (*$BC^2 + AB^2 = AC^2$, By Pythagoras Theorem*)

$\Rightarrow$ $1 + \cot^2\theta = \text{cosec}^2\theta$. **Proved.**

Second method

(ii) $\boxed{1 + \tan^2\theta = \sec^2\theta}$

We have proved that $\sin^2\theta + \cos^2\theta = 1$

Dividing by $\cos^2\theta$ we get

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \quad \tan^2\theta + 1 = \sec^2\theta.$$

(iii) $\boxed{1 + \cot^2\theta = \text{cosec}^2\theta}$

We know that $\sin^2\theta + \cos^2\theta = 1$

Dividing by $\sin^2\theta$, we get

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \text{cosec}^2\theta.$$

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

(i) We know that $\text{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\Rightarrow \mathbf{\sin A} = \sqrt{\frac{1}{1+\cot^2 A}} = \frac{\mathbf{1}}{\sqrt{\mathbf{1+\cot^2 A}}}$$

(ii) $\sec^2 A = 1 + \tan^2 A$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{\mathbf{\cot^2 A + 1}}}{\mathbf{\cot A}}.$$

(iii) $\tan A = \dfrac{1}{\cot A}$

2. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

2. Write all the other trigonometric ratios of ∠ A in terms of sec A.

Since $\sin^2 A + \cos^2 A = 1$, therefore

$$\sin^2 A = 1 - \cos^2 A = \frac{1}{1} - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A} \Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{1}{\sec A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}{\frac{1}{\sec A}} = \frac{\sqrt{\sec^2 A - 1}}{1} = \sqrt{\sec^2 A - 1}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate:

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$$

$$= \frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°} = 1.$$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin 25° \cos(90 - 25°) + \cos 25° \sin(90 - 25°)$

$= \sin^2 25° + \cos^2 25° = 1.$

4.Choose the correct option. Justify your choice.

(i) 9 sec2 A – 9 tan2 A =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ – cosec θ) =

(A) 0 (B) 1 (C) 2 (D) –1

(iii) (sec A + tan A) (1 – sin A) =

(A) sec A (B) sin A (C) cosec A (D) cos A (*i*) $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$

Correct option is (B)

(*ii*) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) = \left(\frac{1}{1} + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(\frac{1}{1} + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right) = \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta\sin\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta\sin\theta} = \frac{2\cos\theta\sin\theta}{\cos\theta\sin\theta} = 2$$

Correct option is (C)

(*iii*) $(\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)\left(\frac{1 - \sin A}{1}\right) = \left(\frac{1 + \sin A}{\cos A}\right)\left(\frac{1 - \sin A}{1}\right)$

$$= \frac{(1)^2 - (\sin A)^2}{\cos A} = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

Correct option is (D)

(*iv*) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

Correct option is (D)

HOME ASSIGNMENT Ex. 8.4 Q. No 1 to 4

AHA

1. Express the ratios cos A, tan A and sec A in terms of sinA

THANKING YOU

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