INTRODUCTION TO TRIGONOMETRY

PPT-9

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

PREVIOUS KNOWLEDGE TEST

- TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = \cos\theta/\sin\theta$

1.$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

2.$\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$

3.$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

- $\sin\theta\ \text{cosec}\ \theta = 1$
- $\Rightarrow \cos\theta \sec\theta = 1$
- $\Rightarrow \tan\theta \cot\theta = 1$

LEARNING OUTCOME

1 . Students will be able to know the Trigonometric Identities.

2. Students will be able to solve the problems involving Trigonometric Identities

3. Students will be able to apply and analyze Trigonometric Identities in solving problems.

4. Students will be able to convert t- ratios in terms of other t-ratios..

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}\,A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \text{cosec}\,A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec}\,A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

(i) We have, $(\text{cosec}\ \theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{LHS} = (\text{cosec}\ \theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

= RHS. Hence, **proved**.

(ii) $\text{LHS} = \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2+2\sin A}{\cos A(1+\sin A)} = \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$= 2\sec A =$ RHS.

Hence, **proved**.

(iii) LHS $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta \times \sin\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos\theta \times \cos\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta + \sin\theta\cos\theta\right)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin\theta\cos\theta+1}{\cos\theta\sin\theta}$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta} + \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$

$$= 1 + \frac{1}{\cos\theta}\frac{1}{\sin\theta}$$

$$= 1 + \sec\theta \operatorname{cosec}\theta = \text{RHS}.$$

Hence, **proved**.

(iv) LHS$= \dfrac{1+\sec A}{\sec A} = \dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = 1+\cos A$

$$= \frac{(1+\cos A)\times(1-\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} = \text{RHS}.$$

Hence, **proved**.

(vi) LHS $= \sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \text{RHS}.$$

Hence, **proved**.

(vii) LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta(\sin^2\theta+\cos^2\theta-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta-\sin^2\theta-\cos^2\theta)}$$

$$= \frac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}.$$

HOME ASSIGNMENT Ex. 8.4 Q. No 5

AHA

1. Prove that sec A (1 – sin A)(sec A + tan A) = 1.

THANKING YOU

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