INTRODUCTION TO TRIGONOMETRY

PPT-10

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 08
CHAPTER NAME : INTRODUCTION TO TRIGONOMETRY

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PREVIOUS KNOWLEDGE TEST

- TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = \cos\theta/\sin\theta$

1.$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

2.$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$

3.$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

- $\sin\theta \operatorname{cosec}\theta = 1$
- $\Rightarrow \cos\theta \sec\theta = 1$
- $\Rightarrow \tan\theta \cot\theta = 1$

LEARNING OUTCOME

1 . Students will be able to know the Trigonometric Identities.

2. Students will be able to solve the problems involving Trigonometric Identities

3. Students will be able to apply and analyze Trigonometric Identities in solving problems.

4. Students will be able to convert t- ratios in terms of other t-ratios..

Trigonometric Identities :

https://youtu.be/3e3k1YH7G40 (10.45)

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}\,A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \text{cosec}\,A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec}\,A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

(v) LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$$

$$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \dfrac{\operatorname{cosec} A + \cot A - (\operatorname{cosec}^2 A - \cot^2 A)}{(\cot A - \operatorname{cosec} A + 1)}$$

$[\because \operatorname{cosec}^2 A = 1 + \cot^2 A \Rightarrow \operatorname{cosec}^2 A - \cot^2 A = 1]$

$$= \dfrac{\operatorname{cosec} A + \cot A - (\operatorname{cosec} A + \cot A) \cdot (\operatorname{cosec} A - \cot A)}{(\cot A - \operatorname{cosec} A + 1)}$$

$$= \dfrac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A + 1)}$$

$= \operatorname{cosec} A + \cot A =$ RHS.

Hence, **proved**.

(viii) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + 2 + \operatorname{cosec}^2 A + \sec^2 A + 2$

$= 1 + 4 + (1 + \cot^2 A) + (1 + \tan^2 A)$

$= 7 + \tan^2 A + \cot^2 A =$ RHS.

Hence, **proved**.

(ix) LHS $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\operatorname{cosec} A - \frac{1}{\operatorname{cosec} A}\right)\left(\sec A - \frac{1}{\sec A}\right)$$

$$= \left(\frac{\operatorname{cosec}^2 A - 1}{\operatorname{cosec} A}\right)\left(\frac{\sec^2 A - 1}{\sec A}\right)$$

$$= \frac{\cot^2 A}{\operatorname{cosec} A} \times \frac{\tan^2 A}{\sec A}$$

$$= \frac{\sin A}{\tan^2 A} \times \cos A \tan^2 A = \sin A \cos A$$

$$\text{RHS} = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{\sin A \cos A}{1}$$

$\because$ LHS = RHS Hence, **proved.**

(x) LHS $= \dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A} = \dfrac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A.$$

$$\text{RHS} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2$$

$$= \frac{(\cos A - \sin A)^2 \times \sin^2 A}{(\sin A - \cos A)^2 \times \cos^2 A}$$

$$= \frac{(\sin A - \cos A)^2 \times \sin^2 A}{(\sin A - \cos A)^2 \times \cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A.$$

$\because$ LHS = RHS Hence, **proved**.

. If $7\sin^2 A + 3\cos^2 A = 4$, show that $\tan A = 1/\sqrt{3}$

. If $7\sin^2 A + 3\cos^2 A = 4$, show that $\tan A = 1/\sqrt{3}$

Given, $7\sin^2 A + 3\cos^2 A = 4$

Dividing both sides by $\cos^2 A$, we get

$$7\tan^2 A + 3 = 4\sec^2 A$$

$$\Rightarrow \quad 7\tan^2 A + 3 = 4(1 + \tan^2 A) \qquad [\because \sec^2\theta = 1 + \tan^2\theta]$$

$$7\tan^2 A + 3 = 4 + 4\tan^2 A$$

$$\Rightarrow \quad 3\tan^2 A = 1$$

$$\Rightarrow \quad \tan^2 A = \frac{1}{3} \quad \Rightarrow \tan A = \frac{1}{\sqrt{3}}$$

Hence proved.

. Prove that: $(\sec\theta + \tan\theta)^2 = \csc\theta + 1/\csc\theta - 1$

. Prove that: (sec θ + tan θ)² = cosec θ +1/cosec θ -1

$$\text{LHS} = (\sec\theta + \tan\theta)^2$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2 = \frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = \frac{1+\sin\theta}{1-\sin\theta}$$

$$= \frac{\left(\frac{1+\sin\theta}{\sin\theta}\right)}{\left(\frac{1-\sin\theta}{\sin\theta}\right)} = \frac{\left(\frac{1}{\sin\theta}+1\right)}{\left(\frac{1}{\sin\theta}-1\right)} = \frac{\operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta - 1} = \text{RHS}$$

. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

. If cos θ + sin θ =√2 cos θ, show that cos θ – sin θ = √2 sin θ.

Here,

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\sin\theta = (\sqrt{2} - 1)\cos\theta$$

$$(\sqrt{2} + 1)\sin\theta = (\sqrt{2} - 1)(\sqrt{2} + 1)\cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

Hence proved

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. If $\sqrt{3} \sin \theta - \cos \theta = 0$ and $0° < \theta < 90°$, find the value of θ.

. If √3 sin θ – cos θ = 0 and 0° < θ < 90°, find the value of θ.

$$\sqrt{3}\sin\theta - \cos\theta = 0$$

$$\Rightarrow \quad \sqrt{3}\sin\theta = \cos\theta \Rightarrow \sqrt{3} = \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad \cot\theta = \sqrt{3} \quad \Rightarrow \theta = 30^\circ$$

. If $6x = \sec\theta$ and $6/x = \tan\theta$, find the value of $9(x^2 - 1/x^2)$

. If 6x = sec θ and 6/x = tanθ, find the value of $9(x^2-1/x^2)$

$$6x = \sec\theta \text{ and } \frac{6}{x} = \tan\theta \Rightarrow x = \frac{\sec\theta}{6} \text{ and } \frac{1}{x} = \frac{\tan\theta}{6}$$

Now,
$$9\left(x^2 - \frac{1}{x^2}\right) = 9\left(\frac{\sec^2\theta}{36} - \frac{\tan^2\theta}{36}\right)$$
$$= \frac{9}{36}(\sec^2\theta - \tan^2\theta) = \frac{1}{4}\times 1 = \frac{1}{4}$$

HOME ASSIGNMENT Ex. 8.4 Q. No 5

AHA

1. . Prove that sec A $(1 - \sin A)(\sec A + \tan A) = 1$.

THANKING YOU

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