ODM EDUCATIONAL GROUP
Changing your Tomorrow

SOME APPLICATIONS OF TRIGONOMETRY
PPT-5

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 09
CHAPTER NAME : SOME APPLICATIONS OF TRIGONOMETRY

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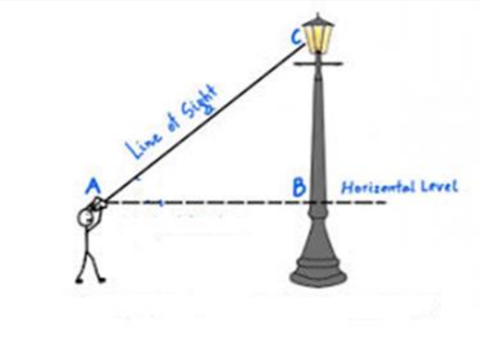
PREVIOUS KNOWLEDGE TEST

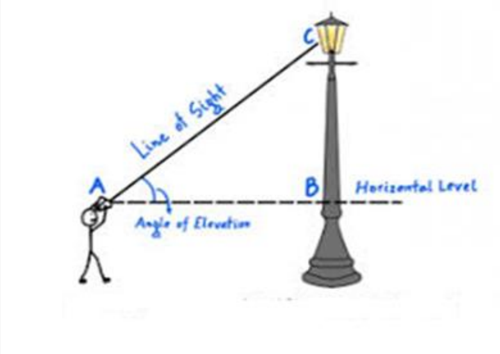
Line of sight: line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer

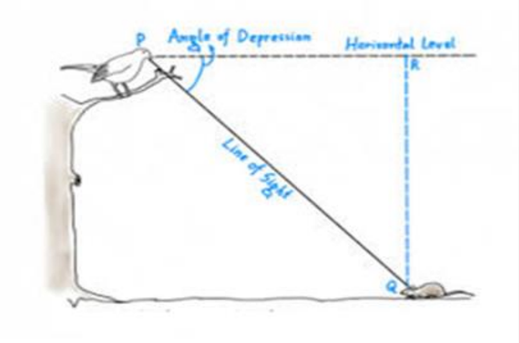
Horizontal level: It is the horizontal line through the eye of the observer

Angle of elevation: The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object.

Angle of depression: The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed







LEARNING OUTCOME

1.Students will be able to know the meaning of line of sight, angle of elevation & angle of depression.

2.Students will be able to analyze word problem and to draw the corresponding figure.

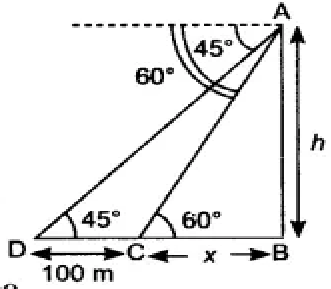
3. Students will be able to apply the knowledge of trigonometry in solving real life problems.

Problem solving on heights and distances: https://youtu.be/8gTCEvlNo-0(10.37)

1. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower.

Let AB be the tower of height h and C and D are positions of the cars.

Let BC = x



In ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3}$

$\Rightarrow$ $x = \frac{h}{\sqrt{3}}$ ...(i)

In ΔABD, $\frac{AB}{BD} = \tan 45° \Rightarrow \frac{h}{100 + x} = 1$

$\Rightarrow$ $h = 100 + x$ ...(ii)

From (i) and (ii), we get

$$h = 100 + \frac{h}{\sqrt{3}} \Rightarrow h - \frac{h}{\sqrt{3}} = 100$$

$$\Rightarrow \sqrt{3}h - h = 100\sqrt{3} \Rightarrow (\sqrt{3} - 1)h = 100\sqrt{3}$$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3} - 1}$$

$$\Rightarrow h = \frac{100\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{100(3+\sqrt{3})}{2}$$

$$= 50 \times (3 + 1.73) = 50 \times 4.73 = 236.5 \text{ m}$$

Hence, the height of the tower is 236.5 m.

2. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A

Let C is the cloud and R is its reflection.

$\angle DAC = 30°$, $\angle DAR = 60°$, let $CD = x$

$\therefore$ Height of the cloud above the lake $= (x + 20)$ m

$\therefore$ $ER = (20 + x)$ m.

Now, in right ΔADC,

$$\frac{CD}{AD} = \tan 30°$$

$$\Rightarrow \frac{x}{AD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AD = \sqrt{3}x \quad ...(i)$$

In right, ΔADR,

$$\frac{DR}{AD} = \tan 60°$$

$$\Rightarrow \frac{DE + ER}{AD} = \sqrt{3}$$

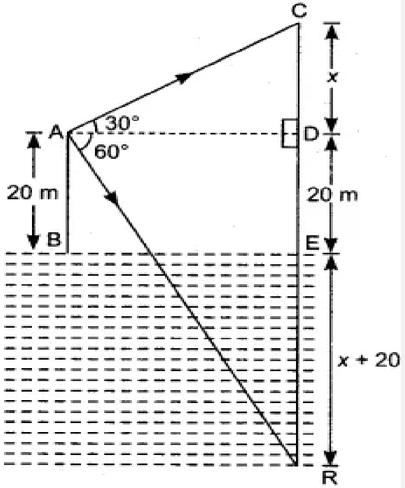
$$\Rightarrow \frac{20 + 20 + x}{\sqrt{3}x} = \sqrt{3} \quad \text{[using (i)]}$$

$$40 + x = 3x \Rightarrow x = 20 \text{ m}$$

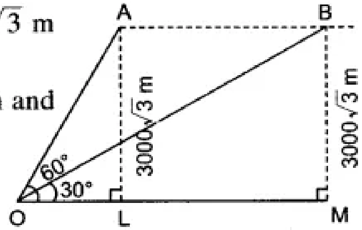
Now, in right ΔADC,

$$\frac{AC}{CD} = \text{cosec } 30° \Rightarrow \frac{AC}{20} = 2 \Rightarrow AC = 40 \text{ m}$$

Hence, the distance of the cloud from A is 40 m.



3.The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of 3000√3 m, find the speed of the aeroplane.



From the point of observation (O), plane is at A, AL = $3000\sqrt{3}$ m and $\angle AOL = 60°$.

After 30 seconds, plane is at B, therefore, BM = $3000\sqrt{3}$ m and $\angle BOM = 30°$.

Distance AB is covered in 30 seconds.

In right-angled triangle OLA,

$$\frac{OL}{AL} = \cot 60°$$

$$\Rightarrow \quad OL = 3000\sqrt{3} \times \frac{1}{\sqrt{3}} = 3000 \text{ m} \qquad ...(i)$$

In right-angled triangle OMB,

$$\frac{OM}{BM} = \cot 30°$$

$$\Rightarrow \quad OM = 3000\sqrt{3} \times \sqrt{3} = 9000 \text{ m} \qquad ...(ii)$$

$$\therefore \quad AB = LM = OM - OL = (9000 - 3000) = 6000 \text{ m} \quad \text{[from } (i) \text{ and } (ii)\text{]}$$

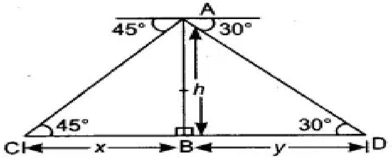
Now, distance covered in 30 s = 6000 m

$\therefore$ Distance covered in 1 hour (3600 s) = $\frac{6000}{30} \times \frac{3600}{1000}$ km = 720 km

$\therefore$ Speed of the aeroplane is 720 km/h.

4.. Two ships are approaching a lighthouse from opposite directions. The angles of depression of the two ships from the top of the lighthouse are 30° and 45°. If the distance between the two ships is 100 m, find the height of the lighthouse

Let AB is lighthouse of height h m. Two ships are represented by C and D where the angles of depression from the lighthouse are 45° and 30° as shown.



Using alternate angles, $\angle ACB = 45°$ and $\angle ADB = 30°$.

Let BC $= x$ and BD $= y$

Given: $$CD = 100 \text{ m}$$

$$\Rightarrow \quad x + y = 100 \quad \ldots(i)$$

In right-angled triangle ABC,

$$\frac{BC}{AB} = \cot 45° \Rightarrow \frac{x}{h} = 1$$

$$\Rightarrow \quad x = h \quad \ldots(ii)$$

In right-angled triangle ABD,

$$\frac{BD}{AB} = \cot 30° \Rightarrow \frac{y}{h} = \sqrt{3}$$

$$\Rightarrow \quad y = \sqrt{3}h \quad \ldots(iii)$$

Putting the values of x and y from (ii) and (iii) in (i), we have

$$h + \sqrt{3}h = 100$$

$$\Rightarrow \quad (\sqrt{3}+1)h = 100 \Rightarrow h = \frac{100}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{100(\sqrt{3}-1)}{3-1} \text{ m} \Rightarrow h = \frac{100(\sqrt{3}-1)}{2}$$

$$= 50(1.732 - 1) \text{ m} = 50 \times 0.732 = 36.6 \text{ m}$$

Hence, height of the lighthouse is 36.6 m.

5.From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are 45° and 60° respectively. Find the height of the tower.

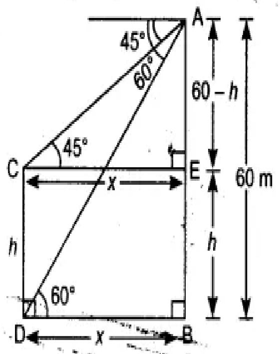
Let AB be 60 m high building and CD be the tower of height h. Angles of depression from top of building to the top and the bottom of the tower are 45° and 60° respectively.

$\therefore \quad \angle ACE = 45^\circ$ and $\angle ADB = 60^\circ$ (using alternate angles)

Let $BD = CE = x$

$BE = CD = h$

$\therefore \quad AE = 60 - h$



In right-angled triangle ABD,

$$\frac{BD}{AB} = \cot 60^\circ$$

$$\Rightarrow \quad \frac{x}{60} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} \quad ...(i)$$

In right-angled triangle AEC,

$$\frac{AE}{CE} = \tan 45^\circ$$

$$\Rightarrow \quad \frac{60-h}{x} = 1$$

$$\Rightarrow \quad 60 - h = x$$

$$\Rightarrow \quad h = 60 - x$$

$$\Rightarrow \quad h = 60 - 20\sqrt{3} \quad \text{[using (i)]}$$

$$\Rightarrow \quad h = 20[3 - \sqrt{3}] = 20[3 - 1.73] = 20 \times 1.27 = 25.4 \text{ m}$$

$\therefore$ Height of the tower is 25.4 m.

HOME ASSIGNMENT Ex 9.1

THANKING YOU

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