4

PROJECTILE MOTION

PROJECTILE MOTION

A body which is in flight through the atmosphere but is not propelled by any fuel is called a projectile. A body or particle moving in atmosphere under effect of gravity only. Motion of projectile is two dimensional motion in a vertical plane.

Ex. Stone thrown in air by a boy, Bullet fired from a gun, Javelin thrown by an athlete, Football kicked by a player, Bomb released from an aeroplane in flight.

Trajectory : Path followed by a projectile is known as trajectory of projectile.

When we consider motion of a projectile, following assumptions are made :

(i) There is no resistance due to air.
(ii) No effect due to curvature of earth.
(iii) No effect due to rotation of earth.
(iv) For all points on trajectory acc. due to gravity g (which is downward) remains same.

Three types of projectile motion :

(i) Oblique Projectile : Body projected at a certain angle with the horizontal.
(ii) Horizontal projectile : Body projected horizontally from a certain height with a certain velocity.
(iii) Projectile motion on inclined plane

PRINCIPLE OF PHYSICAL INDEPENDENCE OF MOTIONS

Motion of projectile is two dimensional motion in a vertical plane. It can be resolved in two motions along horizontal & vertical direction These two motions are independent of each other. This is called principle of physical independence of motions.

At any instant velocity of projectile has two components :

(i) Horizontal Component : No acc. along horizontal (a_x=0) so velocity along horizontal remains unchanged throughout the flight. Horizontal motion is uniform motion.

(ii) Vertical Component : Acceleration due to gravity in downward direction will change the vertical component of velocity continuously throughout the motion. Vertical motion is uniformly accelerated motion.

OBLIQUE PROJECTILE MOTION

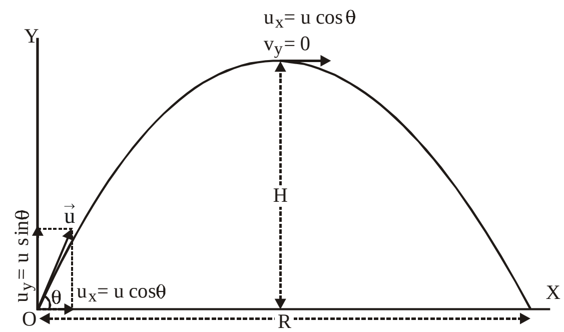
Consider the motion of a body which is projected with initial velocity $\vec{u}$ making an angle θ with the horizontal direction. Let us take X-axis along ground and Y-axis along vertical. $\vec{u}$ can be resolved as

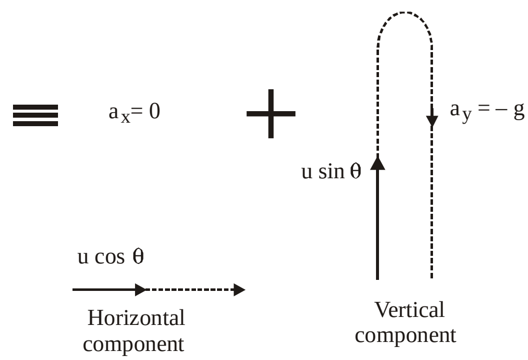
$u_x = u\cos\theta$ (along horizontal)
& $u_y = u\sin\theta$ (along vertical)

motion of body can be resolved into horizontal and vertical motion.

(i) In horizontal direction there is no acc. so it moves with constant velocity $v_x = u_x = u\cos\theta$
So distance traversed in time t is

$$x = u_x t \text{ or } x = (u\cos\theta)\,t \text{ or } t = \frac{x}{u\cos\theta} \quad \text{......(i)}$$





The motion in the vertical direction is the same as that of a body thrown upward with an initial velocity $u_y = u\sin\theta$ and acc. = – g (downward).

So at time t vertical component of velocity

$$v_y = u_y - gt = u\sin\theta - gt \quad \text{.....(ii)}$$

Displacement along y direction

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \quad \text{.....(iii)}$$

Substituting the value of t from eqn. (i) in eqn. (iii)

$$\text{we get,} \quad y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

or $\quad y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}x^2 = ax - bx^2$

This is eqn. of parabola.

The trajectory of projectile is parabolic.

The projectile will rise to maximum height H (where $v_x = u\cos\theta$, $v_y = 0$) and then move down again to reach the ground at a distance R from origin.

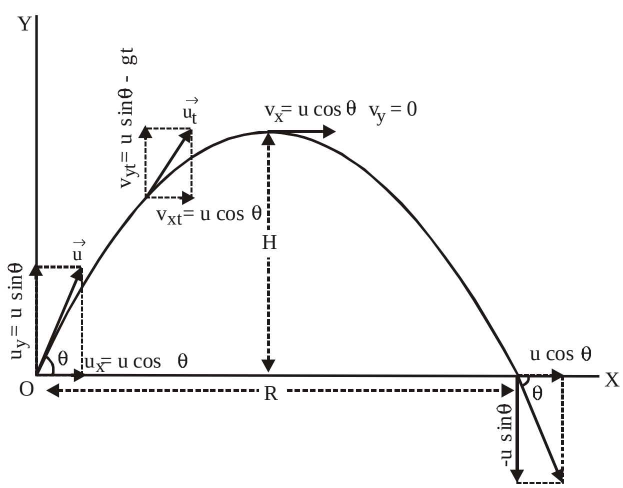
Setting $x = R$ and $y = 0$

(since projectile reaches ground again)

$$0 = R\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot R^2$$

We get $R = \dfrac{2u^2\cos^2\theta}{g}\times\dfrac{\sin\theta}{\cos\theta}$ or $R = \dfrac{2u^2}{g}\cdot\sin\theta\cos\theta$

or **Range** $R = \dfrac{u^2\sin 2\theta}{g}$



If time for upward journey is t

at highest point $v_y = 0$

so, $0 = (u\sin\theta) - gt \qquad (v_y = u_y - gt)$

or $\quad t = \dfrac{u\sin\theta}{g}$

$\therefore \quad T = 2t$ (it will take same time for downward journey)

$\therefore \quad T = \dfrac{2u\sin\theta}{g}$ **Time of flight**

At the highest point $y = H$ and $v_y = 0$

So that , $H = \dfrac{u_y^2}{2g} \qquad [\,v_y^2 = u_y^2 - 2gy\,]$

or $\quad H = \dfrac{u^2\sin^2\theta}{2g}$ **Maximum Height**

we can also determine R as follows, $x = u_x t$

so $\quad R = u_x\,.\,T = (u\cos\theta)\left(\dfrac{2u\sin\theta}{g}\right)$ or $R = \dfrac{u^2\sin 2\theta}{g}$

velocity at time t

$$\vec{v}_t = v_{xt}\,\hat{i} + v_{yt}\,\hat{j} = (u\cos\theta)\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

$$v = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

If $\vec{v}$ makes angle α with horizontal

$$\tan\alpha = \frac{v_{yt}}{v_{xt}} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Note :

(i) Alternative eq^n. of trajectory $y = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

where $R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

(ii) Vertical component of velocity $v_y = 0$, when particle is at the highest point of trajectory.

(iii) Linear momentum at highest point = mu cos θ is in horizontal direction.

(iv) Vertical component of velocity is +ive when particle is moving up.

(v) Vertical component of velocity is –ive when particle is moving down.

(vi) Resultant velocity of particle at time t

$$v = \sqrt{v_x^2 + v_y^2} \text{ at an angle } \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

(vii) Displacement from origin, $s = \sqrt{x^2 + y^2}$

SPECIAL POINTS

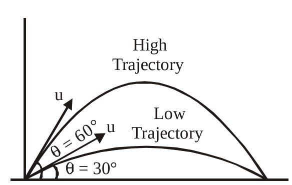
1. In case of projectile motion :

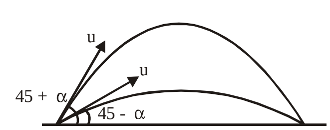
The horizontal component of velocity (u cos θ), acceleration (g) and mechanical energy remains constant. Speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all change. Velocity and K.E. are maximum at the point of projection, while minimum (but not zero) at the highest point.

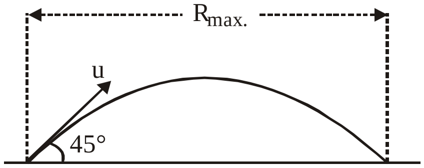
2. If angle of projection is changed from

$\theta \xrightarrow{to} \theta' = (90 - \theta)$, then range

$$R' = \frac{u^2\sin 2\theta'}{g} = \frac{u^2\sin 2(90-\theta)}{g} = \frac{u^2\sin 2\theta}{g} = R$$







Same range : A projectile has same range for angles of projection θ and (90 – θ). But has different time of flight (T), maximum height (H) & trajectories.
Range is also same for

$$\theta_1 = 45° - \alpha \text{ and } \theta_2 = 45° + \alpha. \left[\text{equal } \frac{u^2 \text{os} 2\alpha}{g}\right]$$

For angle of projection θ and (90 – θ). Range is same but maximum height is different.

Maximum height : $H = \frac{u^2 \sin^2\theta}{2g}$

$$\text{and } H' = \frac{u^2 \sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$\frac{H}{H'} = \frac{\sin^2\theta}{\cos^2} = \tan^2\theta$$

$$HH' = \frac{u^4 \sin^2\theta\cos^2\theta}{4g^2} = \frac{R^2}{16} \Rightarrow R = 4\sqrt{HH'}$$

$$H + H' = \frac{u^2\sin^2\theta}{2g} + \frac{u^2\cos^2\theta}{2g} = \frac{u^2}{2g}$$

Time of flight :

$$T = \frac{2u\sin\theta}{g}\ ;\ T' = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\frac{T}{T'} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$TT' = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2R}{g} \Rightarrow TT' \propto R$$

3. For maximum Range

$R = R_{max} \Rightarrow 2\theta = 90°$

for $\theta = 45°,\ R_{max} = \frac{u^2}{g}$ [For $\sin 2\theta = 1 = \sin 90°$ or $\theta = 45°$]

When range is maximum then maximum height reached

$$H = \frac{u^2\sin^2 45}{2g} \text{ (When } R_{max}) \quad \text{or} \quad H = \frac{u^2}{4g}$$

hence maximum height reached (for R_{max})

$$H = \frac{R_{max}}{4}$$

If a body is projected from a place above the surface of earth, then for the maximum range, the angle of projection should be slightly less than 45°. For javelin throw and discus throw, the athlete throws the projectile at an angle slightly less than 45° to the horizontal to achieve the maximum range.

4. For height H to be maximum

$$H = \frac{u^2\sin^2\theta}{2g} = \max \text{ i.e. } \sin^2\theta = 1 \text{ (max) or for } \theta = 90°$$

So that $H_{max} = \frac{u^2}{2g}$

When projected vertically (i.e. at θ = 90°)
In this case Range

$$R = \frac{u^2\sin(2\times 90°)}{g} = \frac{u^2\sin 180°}{g} = 0$$

$$H_{max} = \frac{u^2}{2g} \text{ (For vertical projection) and}$$

$$R_{max} = \frac{u^2}{g} \text{ (For oblique projection with same velocity)}$$

$$\text{so } H_{max} = \frac{R_{max}}{2}$$

If a person can throw a projectile to a maximum distance

(with θ = 45°) $R_{max} = \frac{u^2}{g}$.

The maximum height to which he can throw the projectile

(with θ = 90°) $H_{max} = \frac{R_{max}}{2}$

5. At highest point : Potential energy will be max and equal

to $(PE)_H = mgH = mg.\frac{u^2\sin^2\theta}{2g}$ or $(PE)_H = \frac{1}{2}mu^2\sin^2\theta$.

While K.E. will be minimum (but not zero) and at the highest point as the vertical component of velocity is zero.

$$(KE)_H = \frac{1}{2}mv_H^2 = \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$$

$$\text{so } (PE)_H + (KE)_H = \frac{1}{2}mu^2\sin^2\theta + \frac{1}{2}mu^2\cos^2\theta$$

$$= \frac{1}{2}mu^2 = \text{Total M.E.}$$

So in projectile motion mechanical energy is conserved.

$$\left(\frac{PE}{KE}\right)_H = \frac{\frac{1}{2}mu^2\sin^2\theta}{\frac{1}{2}mu^2\cos^2\theta} = \tan^2\theta$$

6. In case of projectile motion if range R is n times the maximum height H, i.e. R = nH

then $\frac{u^2 \sin 2\theta}{g} = n \cdot \frac{u^2 \sin^2\theta}{2g}$ or $2\cos\theta = \frac{n.\sin\theta}{2}$

or $\tan\theta = \frac{4}{n} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{n}\right)$

If n = 1 i.e. R = H, $\tan\theta = 4 \Rightarrow \theta = 76°$

If $\theta = 45°$, $n = 4 \Rightarrow R = 4H$

7. Weight of a body in projectile motion is zero as it is a freely falling body.

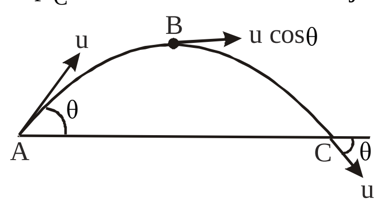
8. **Change in Momentum :**

As in projectile motion velocity in horizontal direction remain constant $(\Delta P)_x = 0$ but in y-direction velocity changes $(\Delta P)_y \neq 0$. Let us write momentum at different position

At A, $\vec{p}_A = m u \cos\theta\ \hat{i} + m u \sin\theta\ \hat{j}$

At B, $\vec{p}_B = m u \cos\theta\ \hat{i}$

At C, $\vec{p}_C = m u \cos\theta\ \hat{i} - m u \sin\theta\ \hat{j}$

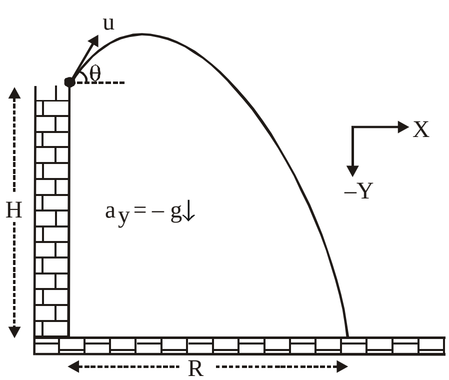


$\Delta\vec{p}_{AB} = \vec{p}_B - \vec{p}_A = -m u \sin\theta\ \hat{j}$

$\Delta\vec{p}_{AC} = \vec{p}_C - \vec{p}_A = -2 m u \sin\theta\ \hat{j}$

OBLIQUE PROJECTILE MOTION FROM HEIGHT H

(A) Projection from a height H at an angle ¯ above horizontal :



$u_x = u\cos\theta$; $u_y = +u\sin\theta$

$x = (u\cos\theta)\,t$; $a_y = -g$

$-H = (u\sin\theta)\,t - \frac{1}{2}gt^2$

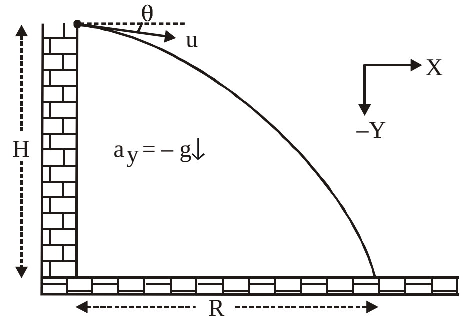
$gt^2 - (2u\sin\theta)\,t - 2H = 0$

$$t = \frac{+2u\sin\theta + \sqrt{(2u\sin\theta)^2 + 8Hg}}{2g}$$

or $t = \frac{u\sin\theta}{g} + \sqrt{\left(\frac{u\sin\theta}{g}\right)^2 + \frac{2H}{g}}$

$R = u\cos\theta \times t$

(B) Projection from a height H at an angle ¯ down horizontal



$u_x = u\cos\theta$; $u_y = -u\sin\theta$

$x = (u\cos\theta)\,t$; $a_y = -g$

$-H = (-u\sin\theta)\,t - \frac{1}{2}gt^2$

$gt^2 + (2u\sin\theta)\,t - 2H = 0$

$$t = \frac{-2u\sin\theta + \sqrt{(2u\sin\theta)^2 + 8Hg}}{2g}$$

or $t = \frac{-u\sin\theta}{g} + \sqrt{\left(\frac{u\sin\theta}{g}\right)^2 + \frac{2H}{g}}$

$R = u\cos\theta \times t$

Example 1 :

A projectile of mass m is projected with velocity v at an angle θ with the horizontal. What is the magnitude of the change in momentum of the projectile after time t ?

Sol. Change in momentum = impulse = force × time = mgt.

Example 2 :

A projectile of mass m is fired with velocity v at an angle θ with the horizontal. What is the change in momentum as it rises to the highest point of the trajectory?

Sol. Change in moment = force × time= $mg \times \frac{v\sin\theta}{g} = mv\sin\theta$

Example 3 :

A ball of mass m is thrown vertically upwards. Another ball of mass 2 m is thrown up making an angle θ with the vertical. Both of them stay in air for the same time. What is the ratio of their maximum heights?

Sol. Since the two bodies are in air for equal interval of time therefore the velocity of projection of first body is equal to the vertical component of the velocity of projection of the second body. So, the maximum heights are the same. The required ratio is 1 : 1.

Example 4 :

What is the angle of projection of an oblique projectile if its range is $\frac{\sqrt{3}\,v^2}{2g}$?

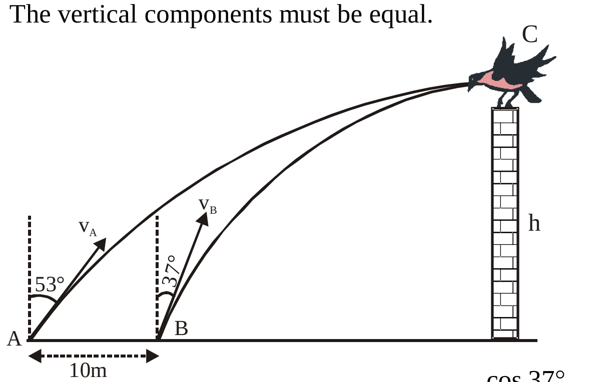
Sol. Comparing the given expression with $R = \frac{v^2 \sin 2\theta}{g}$

we get $\sin 2\theta = \frac{\sqrt{3}}{2}$ or $2\theta = 60°$ or $\theta = 30°$

Example 5 :

Two boys stationed at A and B fire bullets simultaneously at a bird stationed at C. The bullets are fired from A and B at angles of 53° and 37° with the vertical. Both the bullets fire the bird simultaneously. What is the value of v_A if $v_B = 60$ units? Given : tan 37° = 3/4

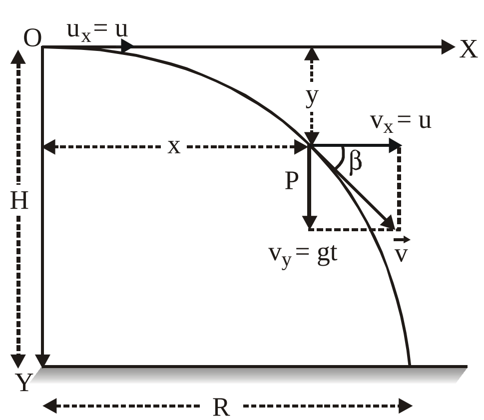
Sol. The vertical components must be equal.



$\therefore \quad v_A \cos 53° = v_B \cos 37°$ or $v_A = v_B \frac{\cos 37°}{\cos(90° - 37°)}$

or $v_A = 60 \cot 37° = \frac{60}{\tan 37°} = \frac{60 \times 4}{3} = 80$ units

HORIZONTAL PROJECTILE MOTION



Suppose a body is thrown horizontally from point O, with velocity u. Height of O from ground = H.

Let X-axis be along horizontal and Y-axis be vertically downwards and origin O is at point of projection as shown in figure.

Let the particle be at P at a time t.

The co-ordinates of P are (x, y).

Distance travelled along X-axis at time t with uniform velocity i.e. velocity of projection and without acceleration.

The horizontal component of velocity $v_x = u$

and horizontal displacement $x = u \cdot t$ ...(i)

displacement along vertical direction is y

to calculate y, consider vertical motion of the projectile

initial velocity in vertical direction $u_y = 0$.

acceleration along y direction $a_y = g$ (acc. due to gravity)

So $v_y = a_y t$ (y comp. of velocity at time t)

or $v_y = gt$ ......(ii)

(as body were dropped from a height)

Resultant velocity at time t is $\vec{v} = u\,\hat{i} + (gt)\,\hat{j}$

$$v = \sqrt{u^2 + (gt)^2}$$

if β is the angle of velocity with X-axis (horizontal)

$$\tan\beta = \frac{gt}{u} \quad \text{and} \quad y = \frac{1}{2}gt^2 \qquad .....(iii)$$

or $y = -\frac{1}{2}g.\left(\frac{x}{u}\right)^2$ [from equation (i) $t = \frac{x}{u}$]

or $y = -\frac{g}{2u^2}.x^2$ or $y = -kx^2$ here $k = \frac{g}{2u^2}$ (k is const.)

This is eqn. of a parabola.

A body thrown horizontally from a certain height above the ground follows a parabolic trajectory till it hits the ground.

(i) Time of flight $T = \sqrt{\frac{2H}{g}}$ [$\therefore y = \frac{1}{2}gt^2$, $T = \sqrt{\frac{2H}{g}}$]

(ii) Range ⇒ horizontal distance covered = R.

$R = u \times$ time of flight

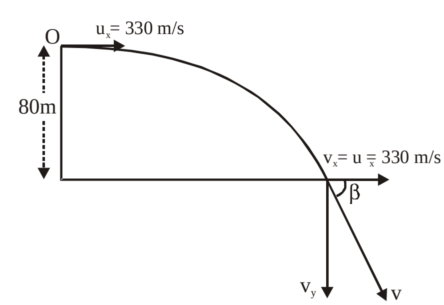
$$R = u \cdot \sqrt{\frac{2H}{g}} \qquad [\because H = \frac{g}{2u^2} R^2]$$

(iii) Velocity when it hits the ground $v_g = \sqrt{u^2 + 2gH}$

Example 6 :

A projectile is fired with a horizontal velocity of 330 ms^{-1} from the top of a cliff 80 m high. How long will it take to strike the ground at the base of the cliff? With what velocity will it strike? Neglect air resistance.

Sol. Let us consider the vertically downward motion.



'u' = 0, $a = +9.8\ m/s^2$, $S = 80\ m, t = ?$

Using, $S = ut + \frac{1}{2}at^2$, we get $80 = \frac{1}{2} \times 9.8\ t^2$

or $t^2 = \frac{160}{9.8} = 16.33 \Rightarrow t = 4.04$ sec

Distance from base $R = ut = 330 \times 4.04 = 1333.20$ m.

Now, $v_y = u_y + a_y t = 9.8 \times 4.04\ ms^{-1} = 39.59$ m/s

Speed $= \sqrt{330^2 + 39.59^2} = 332.37$ m/s

and $\tan\beta = \frac{39.59}{330} = 0.12 \Rightarrow \beta = 6.84°$

Example 7 :

A bomb is dropped from an aeroplane flying horizontal with a velocity of 720 km/h at an altitude of 980 m. After what time, the bomb will hit the ground?

Sol. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{9.8}}$ s $= 10\sqrt{2}$ sec = 14.14 sec

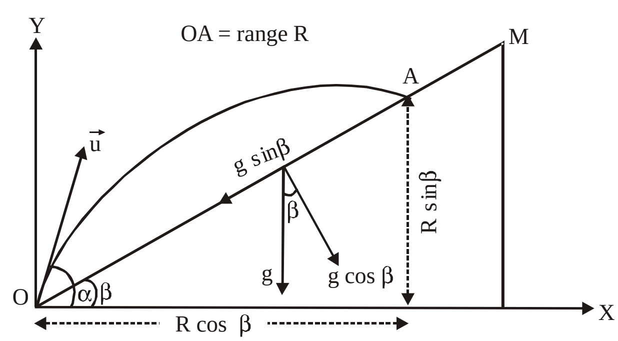
Example 8 :

A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D metre away, then what is the speed of water as it leaves the tank in terms of g, h and D?

Sol. The given problem is the problem of horizontal projectile. The stream of water shall follow the parabolic path.

Now, $t = \sqrt{\frac{2h}{g}}$; $v = \frac{D}{t} = D\sqrt{\frac{g}{2h}}$.

PROJECTION ON AN INCLINED PLANE



$a_x = -g \sin\beta$; $a_y = -g\cos\beta$

$u_x = u\cos(\alpha - \beta)$ $u_y = u\sin(\alpha - \beta)$

or $T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$ is the time of flight

$x = u_x t + \frac{1}{2}a_x t^2$ gives the Range

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

For maximum range $\frac{dR}{d\alpha} = 0 \Rightarrow \alpha = \frac{\pi}{4} + \frac{\beta}{2}$

$$R_{max} = \frac{u^2}{g(1+\sin\beta)}$$

For down the inclined plane replace β by – β.

$$R = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$$

For maximum range, $\alpha = \frac{\pi}{4} - \frac{\beta}{2}$; $R_{max} = \frac{u^2}{g(1-\sin\beta)}$

Example 9:

A ball is thrown from bottom of an incline plane at an angle α from the inclined surface up the plane. Another ball is thrown from a paint on the inclined plane with same speed and at same angle α from the inclined surface down the plane. If in the two cases, maximum height attained by the balls with respect to the inclined surface during projectile motion are h_1 and h_2 then:

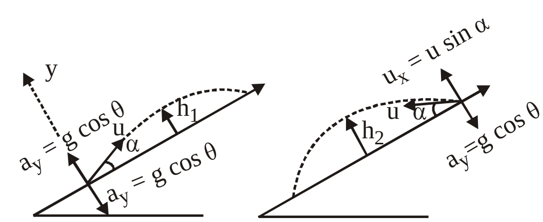
(A) $h_1 > h_2$ (B) $h_1 < h_2$ (C) $h_1 = h_2$

(D) All the three can be possible

Sol. **(C).** For both the particles

$u_y = u\sin\theta$ and $a_y = g\cos\theta$

So, y motion will be similar for both the particles.



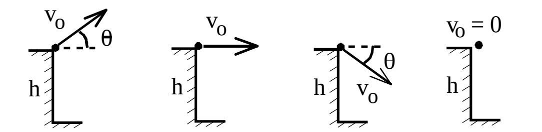
⇒ Max. height and time of flight will be same for the both.

⇒ $h_1 = h_2$.

TRY IT YOURSELF

For Q.1 to Q.5

Projectiles A, B, C, and D are fired at the same time from a height h meters above the ground. With the exception of Projectile D, which is dropped from rest, all the projectiles (i.e., Projectiles A, B, and C) have the same muzzle velocity v_o, (though each is fired at a different angle--see the sketches below and note that the angle defined as θ is the same in all cases). It takes t_1 seconds for Projectile A to get to the top of its flight. It takes t_2 seconds for Projectile D to reach the ground.

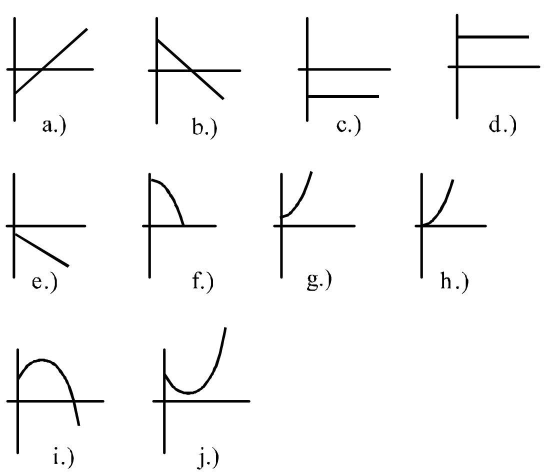


Projectile A Projectile B Projectile C Projectile D

Q.1 At time t_1, Projectile A's:
(A) Velocity will be perpendicular to its acceleration.
(B) Velocity will be $v_o \cos\theta$.
(C) X-component of acceleration will be twice what it was at t = 0.
(D) All of the above responses are true.

Q.2 Projectile A's:
(A) Acceleration is greater on the way up than on the way down.
(B) Velocity changes at the same rate going up as going down.
(C) Y-component acceleration sign is the same as its y-component velocity sign while going up.
(D) Velocity, when at h going upward, will be the same as its velocity when at h coming down.

Q.3 Consider the graphs:



Projectile A's:
(A) Y-component of Position vs. Time graph looks like graph a.
(B) X-component of Position vs. Time graph looks like graph j.
(C) Y-component of Velocity vs. Time graph looks like graph b.
(D) X-component of Velocity vs. Time graph looks like graph c.

Q.4 The time t_2:
(A) Depends only on h and constant(s).
(B) Is the same time it takes Projectile B to hit the ground.
(C) Is more than the time it takes Projectile C to hit the ground, but less than the time it takes Projectile A to hit.
(D) All of the above

Q.5 If h were doubled, Projectile D's:
(A) Time to touch down will double.
(B) Velocity just before touch down will double.
(C) Acceleration just before touch down will double.
(D) None of the above.

Q.6 The trajectory of a projectile is represented by : $\sqrt{3}\,x - gx^2/2$. (y, x in metre) Find the angle of projection, initial speed of projection, range and maximum height.

Q.7 The range of a gun which, fires a shell with muzzle speed V is R. Find the angle of elevation of the gun.

Q.8 A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate
(a) the maximum height,
(b) the time taken by the ball to return to the same level,
(c) the distance from the thrower to the point where the ball returns to the same level.

Q.9 A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})\ ms^{-2}$ (a) At what time is the x- coordinate of the particle 16 m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time ?

Q.10 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

ANSWERS

(1) (D) **(2)** (B) **(3)** (C)
(4) (D) **(5)** (D)
(6) 60º, 2m/s, $\frac{2\sqrt{3}}{g}$m , $\frac{3}{2g}$m **(7)** $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$
(8) (a) 10.0m, (b) 2.9 s, (c) 69m **(9)** 2s, 24m, 21.26m/s
(10) 50m

USE OF RELATIVE CONCEPTS IN PROJECTILE MOTION

(a) If two projectile are projected with speed u_1 and u_2 at an angle of projection θ_1 and θ_2 simultaneously from origin then path of one projectile observe from other projectile will be a straight line.

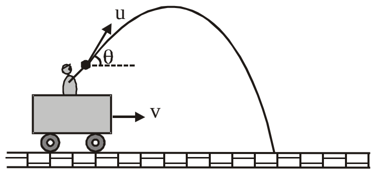
Relative x = $(u_1 \cos\theta_1 - u_2 \cos\theta_2)\,t$

$$\text{Relative y} = (u_1 \sin\theta_1\, t - \frac{1}{2}gt^2) - (u_2 \sin\theta_2\, t - \frac{1}{2}gt^2)$$
$$= (u_1 \sin\theta_1 - u_2 \sin\theta_2)\,t$$
$$\frac{y}{x} = \text{constant} \Rightarrow \text{st. line}$$

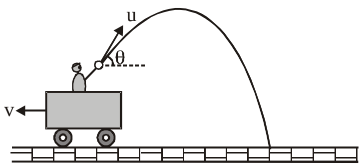
(b) **Projection from a moving body :**
Consider a man who throws a ball from a moving trolley. Let the velocity of ball relative to man be u

$$\vec{V}_{\text{ball, trolley}} = \vec{V}_{\text{ball}} - \vec{V}_{\text{trolley}}$$

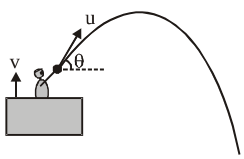
i.e. $\vec{V}_{\text{ball}} = \vec{V}_{\text{ball, trolley}} + \vec{V}_{\text{trolley}}$



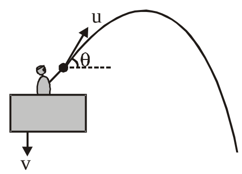
Horizontal component $= u\cos\theta + v$
Vertical component $= u\sin\theta$



Horizontal component $= u\cos\theta - v$
Vertical component $= u\sin\theta$



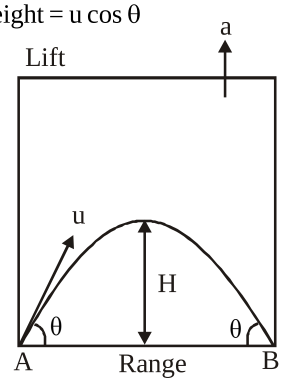
Horizontal component $= u\cos\theta$
Vertical component $= u\sin\theta + v$



Horizontal component $= u\cos\theta$
Vertical component $= u\sin\theta - v$

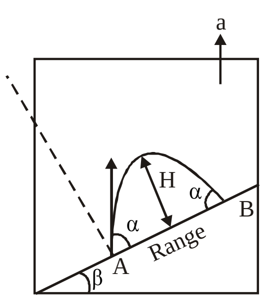
(c) Projectile motion in a lift moving with acceleration a upwards

1. Initial velocity = u (relative to lift), acceleration = a + g (↓) (relative to lift)
2. Velocity at maximum height = $u\cos\theta$
3. $T = \dfrac{2u\sin\theta}{g+a}$
4. Maximum height (H) $= \dfrac{u^2\sin^2\theta}{2(g+a)}$
5. Range $= \dfrac{u^2\sin 2\theta}{g+a}$



Time of flight (T) $= \dfrac{2u\sin\alpha}{(g+a)\cos\beta}$

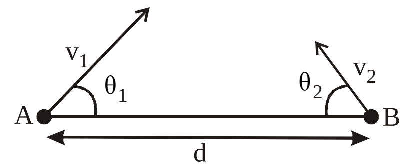
Max. height from incline plane (H) $= \dfrac{u^2\sin^2\alpha}{2(g+a)\cos\beta}$



Maximum distance on incline plane (Range) $= \dfrac{u^2\sin 2\alpha}{(g+a)\cos\beta}$

Example 10 :

Two particles A and B are moving with constant velocities v_1 and v_2. At t = 0, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach and time of collision.



Sol. Velocity of approach is relative velocity along line AB

$$v_{app} = v_1\cos\theta_1 + v_2\cos\theta_1$$

Time of collision, $t = \dfrac{d}{v_{app}} = \dfrac{d}{v_1\cos\theta_1 + v_2\cos\theta_2}$

ADDITIONAL EXAMPLES

Example 1 :

If t_1 be the time taken by a body to clear the top of a building and t_2 be the time spent in air, find the ratio $t_2 : t_1$.

Sol. Total time of flight = 2 (time taken to reach max. height)

$$\Rightarrow t_2 = 2t_1 \Rightarrow \frac{t_2}{t_1} = \frac{2}{1}$$

Example 2 :

When the angle of elevation of a gun are 60º and 30º respectively. The height it shoots are h_1 and h_2 respectively. Find the ratio h_1/h_2.

Sol. For angle of elevation of 60º, we have maximum height

$$h_1 = \frac{u^2\sin^2 60^\circ}{2g} = \frac{3u^2}{8g}$$

For angle of elevation of 30º, we have maximum height

$$h_2 = \frac{u^2\sin^2 30^\circ}{2g} = \frac{u^2}{8g} \;;\; \frac{h_1}{h_2} = \frac{3}{1}$$

Example 3 :

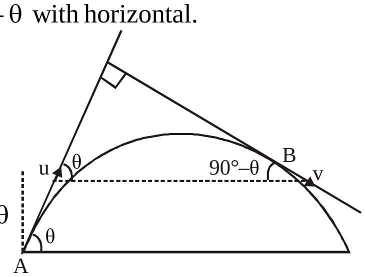
A particle is projectile angle θ with the horizontal with the speed u. Find the velocity perpendicular to initial velocity.

Sol. When particle makes an angle 90° with the initial velocity it will be an angle 90° – θ with horizontal.

Horizontal velocity at A and B should be same

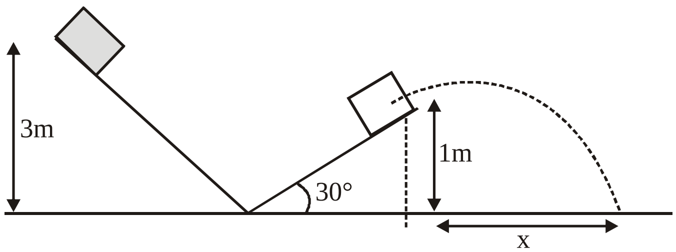
$$u\cos\theta = v\sin\theta$$

$$\Rightarrow v = \frac{u\cos\theta}{\sin\theta} = u\cot\theta$$



Example 4 :

A block slides on a smooth inclined plane as shown in figure find the horizontal distance from the end of the plane when block will strike the ground.



Sol. Change in potential energy = Change in kinetic energy

$$mg[3-1]=\frac{1}{2}mv^2 \Rightarrow v=2\sqrt{g}$$

Vertical component of velocity when block leaves the plane

$$= v\sin 30° = \sqrt{g}$$

$$\because \quad S_y = u_y t + \frac{1}{2}a_y t^2$$

$$\therefore \quad -1=\sqrt{g}t-\frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2-\sqrt{g}t-1=0$$

$$\Rightarrow \quad t^2-\frac{2}{\sqrt{g}}t-\frac{2}{g}=0 \Rightarrow t=\frac{1}{\sqrt{g}}+\frac{\sqrt{3}}{\sqrt{g}}=\frac{\sqrt{3}+1}{\sqrt{g}}$$

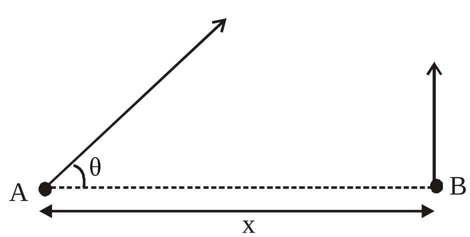
Horizontal displacement = Horizontal velocity × time

$$x=2\sqrt{g}\cos 30° \times t$$

$$\Rightarrow \quad x=2\sqrt{g}\times\frac{\sqrt{3}}{2}\times\left[\frac{\sqrt{3}+1}{\sqrt{g}}\right]=(3+\sqrt{3})\text{ m}=4.73\text{m}$$

Example 5 :

Two particles A and B are projected with the speed v_A = 20m/s and v_B = 10 m/s from the ground as shown in the figure. They collide after 0.5 sec. find the (i) angle θ (ii) value of x.



Sol. If both the particles will be at same height at same time then they will collide.

$y_A = y_B$ [Consider upward direction +ve]

$$\therefore \quad (u_y)_A t-\frac{1}{2}gt^2=(u_y)_B t-\frac{1}{2}gt^2$$

$$\Rightarrow \quad (u_y)_A=(u_y)_B$$

$$\Rightarrow (v_A\sin\theta)=v_B \Rightarrow 20\sin\theta=10 \Rightarrow \sin\theta=\frac{1}{2}\Rightarrow\theta=30°$$

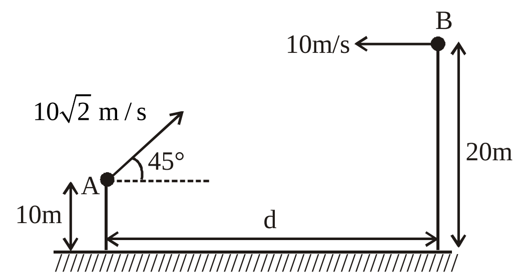
Assume the horizontal distance x travelled by A in 0.5 sec.

$$\therefore \quad x=(u_x)_A t$$

$$x=(20\cos 30°)\,0.5=10\times\frac{\sqrt{3}}{2}=5\sqrt{3}\text{m}$$

Example 6 :

Two particles are projected simultaneously from two towers as shown in the figure. Find the value of d for collision.



Sol. Here acceleration of B relative to A is zero

Therefore, time of collision $t=\dfrac{y_{BA}}{(v_y)_{BA}}$

where y_{BA} = vertical displacement of B wrt to A = 10m

$(v_y)_{BA}$ = vertical component of velocity of B wrt A

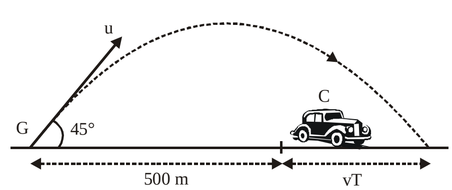
$$=0-(-10\sqrt{2}\sin 45°)=10\text{m/s}\Rightarrow t=\frac{10}{10}=1\text{s}$$

d = relative horizontal displacement of B wrt to A

$$=(v_x)_{BA}\times t=(10+10\sqrt{2}\cos 45°)\times 1=20\text{m}$$

Example 7 :

A gun, kept on a straight horizontal road, is used to hit a car travelling along the same road away from the gun with a uniform sped of 72 km/hr. The car is at a distance of 500 m from the gun, when the gun is fired at an angle of 45° with the horizontal. Find



(a)the distance of the car from the gun when the shell hits it;

(b) The speed of projection of the shell from the gun. ($g = 9.8\text{ m/s}^2$)

Sol. The speed of the car v = 72 × (5/18) = 20 m/s

The time of flight of projectile

$$T = \frac{2u\sin\theta}{g} = \frac{u\sqrt{2}}{g} \quad [\text{as } \theta = 45^\circ] \quad ....(1)$$

and range of projectile

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} \quad ....(2)$$

During the flight of shell the car will cover a distance

$R = 500 + vT$

Substituting the values of T and R form Eqn. (1) and (2) in the above,

$$\frac{u^2}{g} = 500 + \frac{u\sqrt{2}}{g} \times 20 \text{ or } u^2 - 20\sqrt{2}\,u - 4900 = 0$$

or $u = (1/2)\,[20\sqrt{2} \pm \sqrt{(800 + 4 \times 4900)}]$

or $u = 10\,[\sqrt{2} \pm \sqrt{51}]$

As negative sign of u is physically unacceptable,

$u = 10\,[1.414 + 7.141] = 85.56$ m/s

Substituting the above value of u in Eqn. (2)

$$R = \frac{u^2}{g} = \frac{(85.56)^2}{9.8} = 746.9 \text{ m}$$

Example 8 :

Two particle located at a point begin to move with velocities 4m/s and 1 m/s horizontally in opposite directions. Determine the time when their velocity vectors become perpendicular. Assume that the motion takes place in a uniform gravitational field of strength g.

Sol. Velocity of first particle at time $t = 4\hat{i} - gt\,\hat{j}$

Velocity of second particle at time $t = -\hat{i} - gt\,\hat{j}$

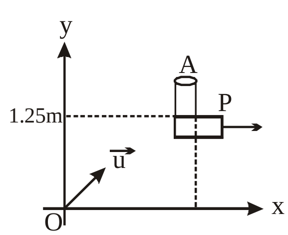
Since the dot product of perpendicular vectors is zero

therefore $(4\hat{i} - gt\,\hat{j}) \cdot (-\hat{i} - gt\,\hat{j}) = 0$

or $4 + g^2t^2 = 0$ or $g^2t^2 = 4$ or $t = 2/g$

Example 9 :

An object A is kept fixed at the point x = 3m and y = 1.25m on a plank P raised above the ground. At time t = 0 the plank starts moving along the +x-direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity $\vec{u}$ as shown.



A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find $\vec{u}$ and the time after which the stone hits the object. Take g = 10 m/s^2

Sol. Let t be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal. According to question, k we have following three conditions :

(i) Vertical displacement of stone is 1.25m.

$$1.25 = (u\sin\theta)\,t - \frac{1}{2}gt^2 \text{ where } g = 10 \text{ m/s}^2$$

or $(u\sin\theta)\,t = 1.25 + 5t^2$ .... (1)

(ii) Horizontal displacement of stone

= 3 + displacement of object A

$\therefore\ (u\cos\theta)\,t = 3 + \frac{1}{2}at^2$, where a = 1.5 m/s^2

or $(u\cos\theta)\,t = 3 + 0.75t^2$ .... (2)

(iii) Horizontal component of velocity (of stone) = vertical component (because velocity vector is inclined at 45° with horizontal)

$\therefore\ (u\cos\theta) = gt - (u\sin\theta)$ .... (3)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion.

Therefore, $gt > u\sin\theta$ in upward motion $u\sin\theta > gt$)

Multiplying eq. (3) with t we can write :

or $(u\cos\theta)\,t + (u\sin\theta)\,t = 10t^2$ .... (4)

Now, eqs (4), (2) and (1) gives

$4.25t^2 - 4.25 = 0$ or $t = 1$s

Substituting t = 1s in eq. (1) and (2), we get

$u\sin\theta = 6.25$ m/s or $u_y = 6.25$ m/s

and $u\cos\theta = 3.75$ m/s or $u_x = 3.75$ m/s

Therefore, $\vec{u} = u_x\hat{i} + u_y\hat{j}$ m/s

or $\vec{u} = (3.75\,\hat{i} + 6.25\hat{j})$ m/s

Example 10 :

Is it important in the long jump that how much height you take for jumping.

Sol. It is important in the long jump how high a person jumps.

As $h = \dfrac{u^2\sin^2\theta}{2g}$

hence $\dfrac{u^2}{g} = \dfrac{2h}{\sin^2\theta}$

and range $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2h}{\sin^2\theta} \times \sin 2\theta = 4h\cot\theta$

i.e. the range of jump is determined by initial speed u and angle θ or height h and angle of projection θ.

Example 11 :

A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60º to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is –

Sol. **5.** $T = \dfrac{2u\sin\theta}{g}$

$$T = \frac{2\times 10\times\sqrt{3}}{10\times 2} = \sqrt{3}\text{ sec} \ ; \ R = u\cos\theta.T - \frac{1}{2}aT^2$$

$$1.15 = 10\times\frac{1}{2}\sqrt{3} - \frac{1}{2}a(\sqrt{3})^2$$

$$\frac{3}{2}a = 5\sqrt{3} - 1.15 \ ; \ \frac{3a}{2} = 8.65 - 1.15 = 7.5$$

$$a = 7.5\times\frac{2}{3} \approx 5 \text{ m/sec}^2$$

Example 12 :

If a projectile has a constant initial speed and angle of projection, find the relation between the change in the horizontal range due to change in acceleration due to gravity.

Sol. Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

Differentiating t w.r.t. we have

$$\frac{dR}{dg} = -\frac{u^2}{g^2}\sin 2\theta \qquad [\because \ u \text{ and } \theta \text{ are constant}]$$

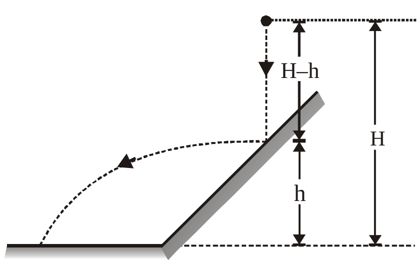
or $dR = -\dfrac{u^2\sin 2\theta}{g^2}\dfrac{dg}{g} = -R\dfrac{dg}{g}$ or $\dfrac{dR}{R} = -\dfrac{dg}{g}$

Example 13 :

A body falling freely from a given height H hits an inclined plane in his path at a height 'h'. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground ?

Sol. Time taken by the body to strike the inclined plane

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$



After impact the velocity becomes horizontal

so time taken to reach the ground $t_2 = \sqrt{\dfrac{2h}{g}}$

Total time of motion

$$t = t_1 + t_2 = [\sqrt{h} + \sqrt{(H-h)}]\sqrt{\frac{2}{g}}$$

For t to be maximum $\left(\dfrac{dt}{dh}\right) = 0$

i.e., $\dfrac{d}{dh}[h^{1/2} + (H-h)^{1/2}]\sqrt{\dfrac{2}{g}} = 0$

or $\dfrac{1}{2}h^{-1/2} + \dfrac{1}{2}(H-h)^{1/2}(-1) = 0$

or $h = H - h$, i.e., $\dfrac{h}{H} = \dfrac{1}{2}$

Example 14 :

A ball rolls horizontal off the top of a stairway with a speed of 1.52 m/s. The steps are 20.3 cm. high and 20.3 cm. wide which step does the ball hit first ?

Sol. Let h be the height of a step and w be the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between (n – 1) w and nw. Take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway. Take the y axis to be positive in the upward direction and the x axis to be horizontal.

The coordinates of the ball at time t are given by

$$x = v_{0x}\,t \text{ and } y = -\frac{1}{2}gt^2.$$

Equate y to – nh and solve for the time to reach the level

step n : $t = \sqrt{\dfrac{2nh}{g}}$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52\text{m/s})\sqrt{\frac{2n\,(0.203\text{m})}{9.8\text{ m/s}^2}} = (0.309\text{m})\sqrt{n}.$$

Try values of n until you find one for which x/w is less than n but greater than n – 1. For n = 1, x = 0.039m and x/w = 1.52. This is greater than n. For n = 2, x = 0.437m and x/w = 2.15. This is also greater than.

For n = 3, x = 0.535m and x/w = 2.64. This is less than n and greater than n – 1. The ball hits the third step.

QUESTION BANK

CHAPTER 4 : PROJECTILE MOTION

EXERCISE - 1 [LEVEL-1]

Choose one correct response for each question.

PART - 1 : OBLIQUE PROJECTILE MOTION

Q.1 The path followed by a body projected along y axis is given as by y = $\sqrt{3}$ x – (1/2) x^2. If g = 10 m/s^2 then the initial velocity of projectile will be- (x and y are in m)
(A) 3 $\sqrt{10}$ m/s (B) 2 $\sqrt{10}$ m/s
(C) 10 $\sqrt{3}$ m/s (D) 10 $\sqrt{2}$ m/s

Q.2 When the angle of elevation of a gun are 60º and 30º respectively. The height it shoots are h_1 and h_2 respectively, h_1/h_2 equals to-
(A) 3/1 (B) 1/3
(C) 1/2 (D) 2/1

Q.3 The height y and the distance x along the horizontal at plane of the projectile on a certain planet (with no surrounding atmosphere) are given by y = (8t – 5t^2) metre and x = 6t metre where t is in seconds. The velocity with which the projectile is projected is-
(A) 8 m/s (B) 6 m/s
(C) 10 m/s (D) Data is insufficient

Q.4 A body is thrown at an angle 30º to the horizontal with the velocity of 30 m/s. After 1 sec, its velocity will be (in m/s) (g = 10 m/s^2)
(A) 10 $\sqrt{7}$ (B) 700 $\sqrt{10}$
(C) 100 $\sqrt{7}$ (D) $\sqrt{10}$

Q.5 A ball thrown by one player reaches the other in 2 sec. The maximum height attained by the ball above the point of projection will be about-
(A) 2.5 m (B) 5 m
(C) 7.5 m (D) 10 m

Q.6 Kalpit and Mukesh are playing with two different balls of masses m and 2m respectively. If Kalpit throws his ball vertically up and Mukesh at an angle θ, both of them stay in our view for the same period. The height attained by the two balls are in the ratio-
(A) 2 : 1 (B) 1 : 1
(C) 1 : cos θ (D) 1 : sec θ

Q.7 A projectile is thrown at angle θ and (90° – θ) from the same point with same velocity 98 m/s. The heights attained by them, if the difference of heights is 50 m will be- (in m)
(A) 270, 220 (B) 300, 250
(C) 250, 200 (D) 200, 150

Q.8 A particle is projected with a velocity u so that its horizontal range is twice the greatest height attained. The horizontal range is-
(A) u^2/g (B) $2u^2/3g$
(C) $4u^2/5g$ (D) $u^2/2g$

Q.9 A particle is projected such that horizontal range and vertical height are same. Then the angle of projection is
(A) $\tan^{-1}$ (4) (B) $\tan^{-1}$ (1/4)
(C) π/4 (D) π/3

Q.10 A projectile at an angle 30º from the horizontal has range R. If the angle of projection at the same initial velocity be 60º, then find the range.
(A) R (B) 2R
(C) R/2 (D) 4R

Q.11 If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will
(A) Remain the same (B) Be doubled
(C) Be quadrupled (D) Be halved

Q.12 The range of a projectile for a given initial velocity is maximum when the angle of projection is 45°. The range will be minimum, if the angle of projection is
(A) 90° (B) 180°
(C) 60° (D) 75°

Q.13 At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
(A) Perpendicular to each other
(B) Parallel to each other
(C) Inclined to each other at an angle of 45°
(D) Antiparallel to each other

Q.14 A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be
(A) R/6 (B) 6R
(C) R/36 (D) 36R

Q.15 A gun is aimed at a target in a line of its barrel. The target is released and allowed to fall under gravity at the same instant the gun is fired. The bullet will
(A) Pass above the target
(B) Pass below the target
(C) Hit the target
(D) Certainly miss the target

Q.16 The equation of motion of a projectile are given by x = 36 t metre and 2y = 96 t – 9.8 t^2 metre. The angle of projection is
(A) $\sin^{-1}$ (4/5) (B) $\sin^{-1}$ (3/5)
(C) $\sin^{-1}$ (4/3) (D) $\sin^{-1}$ (3/4)

PART - 2 : HORIZONTAL PROJECTILE MOTION

Q.17 Savita throws a ball horizontally with a velocity of 8 m/s from the top of the her building. The ball strikes to her brother Sudhir playing at 12m away from the building. What is the height of the building ?
(A) 11m (B) 10 m
(C) 8 m (D) 7 m

Q.18 From the top of a tower 19.6 m high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, then the initial velocity of the ball is
(A) 9.8 ms^{-1} (B) 4.9 ms^{-1}
(C) 14.7 ms^{-1} (D) 2.8 ms^{-1}

Q.19 Two paper screens A and B are separated by 150 m. A bullet pierces A and then B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is –
(A) $100\sqrt{3}$ m/s (B) $200\sqrt{3}$ m/s
(C) $300\sqrt{3}$ m/s (D) $500\sqrt{3}$ m/s

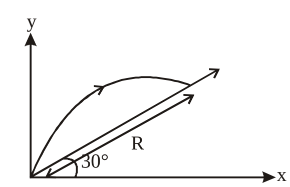
Q.20 An aeroplane is flying at a constant horizontal velocity of 600 km/hr at an elevation of 6 km towards a point directly above the target on the earth's surface. At an appropriate time, the pilot releases a ball so that it strikes the target at the earth. The ball will appear to be falling
(A) On a parabolic path as seen by pilot in the plane.
(B) Vertically along a straight path as seen by an observer on the ground near the target.
(C) On a parabolic path as seen by an observer on the ground near the target.
(D) On a zig-zag path as seen by pilot in the plane.

Q.21 A body is thrown horizontally from the top of a tower of height 5 m. It touches the ground at a distance of 10 m from the foot of the tower. The initial velocity of the body is (g = 10 ms^{-2})
(A) 2.5 ms^{-1} (B) 5 ms^{-1}
(C) 10 ms^{-1} (D) 20 ms^{-1}

Q.22 A particle (P) is dropped from a height and another particle (Q) is thrown in horizontal direction with speed of 5 m/sec from the same height. The correct statement is
(A) Both particles will reach at ground simultaneously.
(B) Both particles will reach at ground with same speed.
(C) Particle (P) will reach at ground first with respect to particle (Q).
(D) Particle (Q) will reach at ground first with respect to particle (P).

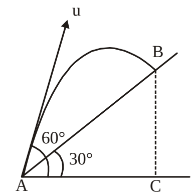
PART - 3 : PROJECTILE MOTION ON AN INCLINED PLANE

Q.23 Initial velocity is 10 m/sec and angle of projection is 60°, find range R



(A) $\frac{15\sqrt{3}m}{2}$ (B) $\frac{40}{3}$m
(C) $5\sqrt{3}$m (D) $\frac{20}{3}$m

Q.24 Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to –



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$
(C) $\sqrt{3}ut$ (D) 2ut

Q.25 A particle is thrown at an angle β with vertical. It reaches a maximum height H. Then the time taken to reach the highest point of its path is –
(A) $\sqrt{\frac{H}{g}}$ (B) $\sqrt{\frac{2H}{g}}$ (C) $\sqrt{\frac{H}{2g}}$ (D) $\sqrt{\frac{2H}{g\cos\beta}}$

Q.26 Two stones A and B are projected with the same velocity at angles of projection 20° and 70° respectively. If H_A and H_B be the horizontal ranges, then –
(A) $H_A > H_B$ (B) $H_A < H_B$
(C) $H_A = H_B$ (D) None of these

Q.27 A cricketer can throw a ball to a maximum horizontal distance of 100 m. The speed with which he throws the ball is (to the nearest integer)
(A) 30 ms^{-1} (B) 42 ms^{-1}
(C) 32 ms^{-1} (D) 35 ms^{-1}

PART - 4 : MISCELLANEOUS

Q.28 A projectile has the same range R for two angles of projection. If t_1 and t_2 are the times of flight in the two cases, then.
(A) $t_1t_2 \propto R$ (B) $t_1t_2 \propto R^2$
(C) $t_1t_2 \propto 1/R$ (D) $t_1t_2 \propto 1/R^2$

Q.29 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ?
(A) 150.5m (B) 160m
(C) 170.4m (D) 145.2m

Q.30 A stone is projected from the ground with velocity 50 m/s at an angle of 30º. It crosses a wall after 3 sec. How far beyond the wall the stone will strike the ground (g = 10 m/sec^2)
(A) 50.5m (B) 60m
(C) 70.4m (D) 86.6m

Q.31 Two boys stationed at A and B fire bullets simultaneously at a bird stationed at C. The bullets are fired from A and B at angles of 53° and 37° with the vertical. Both the bullets fire the bird simultaneously. What is the value of v_A if v_B = 60 units?
Given : tan 37° = 3/4
(A) 80 (B) 90
(C) 100 (D) 70

Q.32 A particle projected from the origin (x = y = 0) moves in xy plane with a velocity v = $2\hat{i} + 4x\hat{j}$, where $\hat{i}$ & $\hat{j}$ are the unit vectors along x and y axis. The equation of the motion of the particle is –
(A) $y = x^2$ (B) $y = 2x^2$
(C) $y = x^2/2$ (D) $y = x^2/4$

Q.33 The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter, where t is in second. The velocity with which the projectile is projected is-

(A) 8 m/sec (B) 6 m/sec
(C) 10 m/sec (D) Not obtainable from the data

Q.34 A cannon ball has the same range R on a horizontal plane for two angles of projection. If h_1 and h_2 are the greatest heights in the two paths for which this is possible, then

(A) $R = h_1 h_2$ (B) $R = 4\sqrt{h_1 h_2}$
(C) $R = \sqrt[3]{h_1 h_2}$ (D) $R = (h_1 h_2)^{\frac{1}{4}}$

Q.35 A very broad elevator is going up vertically with a constant acceleration of 2 m/s^2. At the instant when its velocity is 4 m/s a ball is projected from the floor of the lift with a speed of 4 m/s relative to the floor at an elevation of 30°. The time taken by the ball to return the floor is ($g = 10\ m/s^2$)

(A) (1/2) s (B) (1/3) s
(C) (1/4) s (D) 1s

Q.36 An astronaut in a strange planet observe that he can jump a maximum horizontal distance of 2m, if his initial speed is 6 m/s. What is the acceleration due to gravity of the planet?

(A) 3 m/s^2 (B) 9 m/s^2
(C) 18 m/s^2 (D) 36 m/s^2

Q.37 A projectile is thrown with velocity U = 20m/s ± 5% at an angle 60°. If the projectile falls back on the ground at the same level then which of following can not be a possible answer for range. [Consider $g = 10 m/s^2$]

(A) 39.0 m (B) 37.5 m
(C) 34.6 m (D) 32.0 m

Q.38 A stone projected at angle 53° attains maximum height 25m during its motion in air. Then its distance from the point of projection where it will fall is –

(A) $\frac{400}{3}$ m (B) 50 m
(C) 60m (D) 75m

Q.39 A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level round through the foot of the tower at a distance x from the tower. The value of x is –

(A) h (B) h/2
(C) 2h (D) 2h/3

Q.40 If the range of the projectile be R, then the potential energy will be maximum after the projectile has covered (from start) a distance equal to-

(A) R (B) R/2
(C) R/4 (D) R/8

Q.41 Ratio of the ranges of the bullets fired from a gun at angle $\theta, 2\theta$ & 4θ is found in the ratio x : 2 : 2, then the value of x will be (Assume same speed of bullets) –

(A) 2 (B) $\sqrt{3}$
(C) 1 (D) None of these

Q.42 A ball of mass m is thrown vertically upwards. Another ball of mass 2m is thrown at an angle θ with the vertical with some velocity. Both of them stay in air for same period of time. The heights attained by the two balls are in the ratio of-

(A) 2 : 1 (B) 1 : cos θ
(C) 1 : 1 (D) cos θ : 1

Q.43 An arrow is shot into the air on a parabolic path to a target. Neglecting air resistance, at its highest point –

(A) both velocity and acceleration vectors are horizontal
(B) the acceleration vector is zero but not the velocity
(C) the velocity and acceleration vectors are both zero
(D) the upward component of velocity is zero but not the acceleration

Q.44 A jogger runs with constant velocity v through a forest of coconut trees. A coconut starts to fall from a height h when the jogger is directly below it. How far behind the jogger will the coconut land ?

(A) $\sqrt{\frac{2hv^2}{g}}$ (B) $\sqrt{\frac{hv^2}{2g}}$
(C) $\frac{gh^2}{2v^2}$ (D) $\frac{2gh^2}{v^2}$

Q.45 A body is projected horizontally from the top of a tower with initial velocity 18m/s. It hits the ground at angle 45°. What is the vertical component of velocity when it strikes the ground ?

(A) $18\sqrt{2}$ m/s (B) 18 m/s
(C) $9\sqrt{2}$ m/s (D) 9 m/s

Q.46 A cricketer hits a ball with a velocity 25 m/s at 60º above the horizontal. How far above the ground it passed over a fielder 50 m from the bat (assume the ball is struck very close to the ground)-

(A) 8.2 m (B) 9.0 m
(C) 11.6 m (D) 12.7 m

Q.47 A particle is projected from the ground with an initial velocity of 20m/s at an angle of 30° with horizontal. The magnitude of change in velocity in time interval of 0.5 sec starting from instant of projection is –
(Neglect air friction and take $g = 10\ m/s^2$)

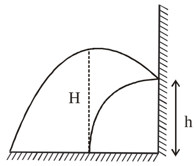
(A) 5 m/s (B) 2.5 m/s
(C) 2 m/s (D) 4 m/s

Q.48 A body of mass m is projected at an angle of 45º with the horizontal with velocity v. If air resistance is negligible, then total change in momentum when it strikes the ground is-

(A) 2 mv (B) $\sqrt{2}$ mv
(C) mv (D) mv/$\sqrt{2}$

EXERCISE - 2 [LEVEL-2]

Q.1 A stone is projected from a horizontal plane. It attains maximum height 'H' & strikes a stationary smooth wall & falls on the ground vertically below the maximum height. Assume the collision to be elastic, the height of the point on the wall where ball will strike is –

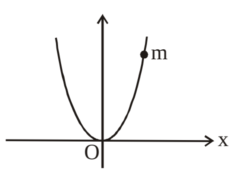


(A) H/2 (B) H/4
(C) 3H/4 (D) None of these

Q.2 A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7m away and level with the rifle. How high above the target must the rifle barrel be pointed so that the bullet hits the target ?
(A) 4.84 cm. (B) 2.12 cm.
(C) 3.14cm. (D) 5.34cm.

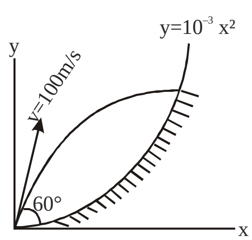
Q.3 A particle is projected from a point (0,1) on Y-axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It fell on ground along x axis in 1 sec.
Taking g = 10 m/s^2 and all coordinate in metres. Find the X-coordinate where it fell.
(A) (3, 0) (B) (4, 0)
(C) (2,0) (D) $(2\sqrt{5}, 0)$

Q.4 A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is :



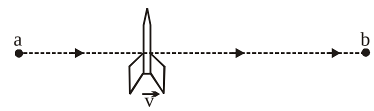
(A) g/2 (B) $\sqrt{3}g/2$
(C) $g/\sqrt{2}$ (D) $g/\sqrt{5}$

Q.5 A projectile is fired at an angle of 60º with muzzle velocity 100 m/s as shown. At what elevation y does it strike the hill whose equation has been estimated as
$y = 10^{-3} x^2$. Neglect air friction. (Take g = 10 m/s^2)

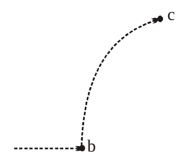


(A) $\sqrt{3}$ km (B) $\frac{1}{\sqrt{3}}$ km.
(C) 3 km (D) $\frac{1}{3}$ km

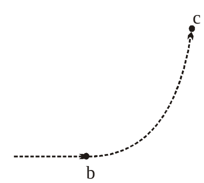
Q.6 A rocket drifting side ways in outer spaces from position 'a' to position 'b' with constant velocity. At 'b', the rocket's engine starts to produce a constant thrust at right angles to line 'ab'. The engine turns off again as the rocket reaches some point "c". Assume that rocket is subjected to no other forces. Choose the incorrect statement –



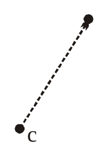
(A) The path of rocket from point b to c will be



(B) The path of rocket from point b to c will be

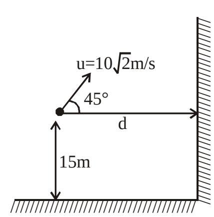


(C) The path of rocket beyond c will be



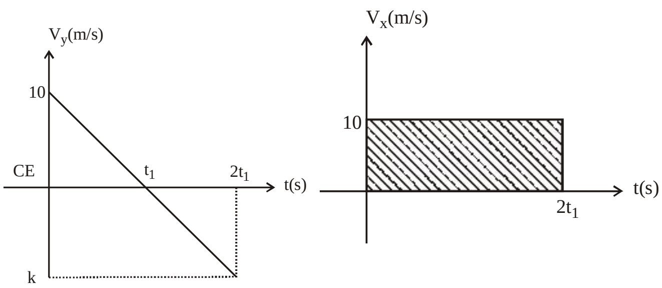
(D) The speed continuously increase from b to c.

Q.7 A small ball is thrown from a height of 15m above ground and at a horizontal distance d from a vertical wall. The ball first hits the wall and then strikes the ground and then it flies back to its initial position of throwing. Take both collisions to be perfectly elastic and neglect friction. The initial speed of the ball is $10\sqrt{2}$ m/s and angle of projection is 45° with the horizontal as shown. Find the horizontal distance of point of throwing from the wall 'd' in meters. (Neglect air resistance and take g = 10 m/s^2)



(A) 20m (B) 10m
(C) 40m (D) 30m

Q.8 A projectile is thrown from the origin in x-y plane, where x-axis is along the ground and y-axis is the vertically upwards. The vertical velocity and the horizontal velocity vary with respect to time according to the graphs shown. Accelerating due to gravity is g. What is the value of t_1

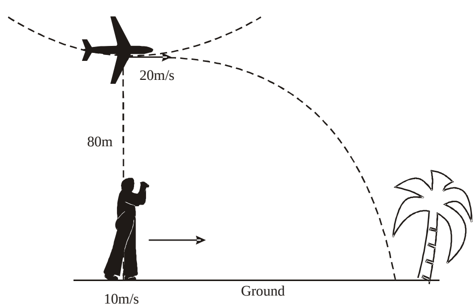


(A) 10/g (B) 20/g
(C) 30/g (D) 10/2g

Q.9 In the above question, what is the value of k –
(A) 10 (B) – 10
(C) 20 (D) – 20

Q.10 In the above question, what is the initial angle of projection
(A) 45° (B) 75°
(C) 60° (D) 30°

Q.11 A bomber plane moving at a horizontal speed of 20m/s releases a bomb at a height of 80m above ground as shown. At the same instant a Hunter starts running from a point below it, to catch the bomb at 10 m/s. After two seconds he realized that he cannot make it, he stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, bullet will hit it. What should be the firing speed of bullet. (Take $g = 10\ m/s^2$)



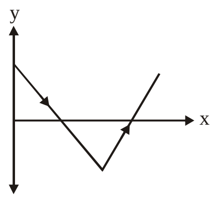
(A) 10 m/s (B) $20\sqrt{10}$ m/s
(C) $10\sqrt{10}$ m/s (D) None of these

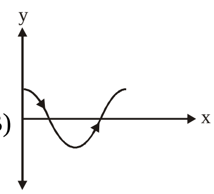
Q.12 An object is moving in the xy plane with the position as a function of time given by $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$. Point O is at $\vec{r} = 0$. The distance of object from O is definitely decreasing when –
(A) $v_x > 0, v_y > 0$ (B) $v_x < 0, v_y < 0$
(C) $xv_x + yv_y < 0$ (D) $xv_x + yv_y > 0$

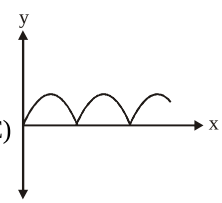
Q.13 A particle starts moving at t = 0 in x-y plane such that its coordinates (mm) with time (in sec.) as x = 2t and y = 5 sin (2t). If magnitude of its acceleration a, then at all the times –
(A) $a \propto x$ (B) $a \propto \sqrt{x^2 + y^2}$
(C) $a \propto y$ (D) a = 0

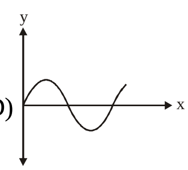
Q.14 In the above question, maximum speed of the particle is
(A) 12 m/s (B) $\sqrt{29}$ m/s
(C) $2\sqrt{26}$ m/s (D) 10 m/s

Q.15 In the above question, the path of the particle will be –

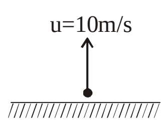
(A) 

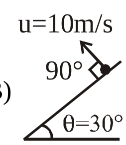
(B) 

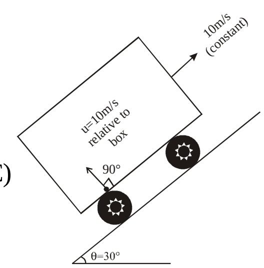
(C) 

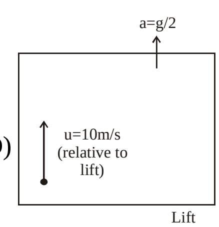
(D) 

Q.16 In which of the following cases the time of flight is min –

(A) 

(B) 

(C) 

(D) 

Q.17 Velocity of a stone projected, 2 second before it reaches the maximum height, makes angle 53° with the horizontal then the velocity at highest point will be (Neglect air friction and take $g = 10\ m/s^2$)
(A) 20 m/s (B) 15 m/s
(C) 25 m/s (D) 80/3 m/s

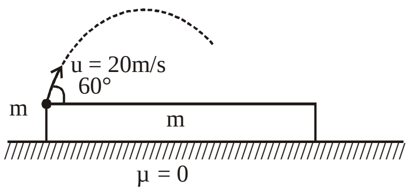
Q.18 If a particle is projected with speed u from ground at an angle θ with horizontal ,then radius of curvature of a point where velocity vector is perpendicular to initial velocity vector is given by –

(A) $\frac{u^2\cos^2\theta}{g}$ (B) $\frac{u^2\cot^2\theta}{g\sin\theta}$ (C) $\frac{u^2}{g}$ (D) $\frac{u^2\tan^2\theta}{g\cos\theta}$

EXERCISE - 3 (NUMERICAL VALUE BASED QUESTIONS)

NOTE : The answer to each question is a NUMERICAL VALUE.

Q.1 A particle of mass m is projected at an angle of 60° with a velocity of 20 m/s relative to the ground from a plank of same mass m which is placed on a smooth surface initially plank was at rest. The minimum length of the plank for which the ball will fall on the plank itself is $10 \times A \times \sqrt{3}m$. Find the value of A.



Q.2 Distance between a frog and an insect on a horizontal plane is 10m. Frog can jump with a maximum speed of $\sqrt{10}$ m / s. Find the minimum number of jumps required by the frog to catch the insect.

Q.3 A truck starts from origin, accelerating with 'a' m/sec^2 in positive x-axis direction. After 2 seconds a man standing at the starting point of the truck projected a ball at an angle 30° with velocity v m/s.The relation between 'a' and 'v' is $\frac{\sqrt{3}v^2}{g} = a\left(A + \frac{v}{g}\right)^2$ such that ball hits the truck. (assume truck is moving on horizontal plane and man projected the ball from the same horizontal level of truck). Find the value of A.

Q.4 Three stones A, B and C are simultaneously projected from same point with same speed. A is thrown upwards, B is thrown horizontally and C is thrown downwards from a building. When the distance between stone A and C becomes 10m, then distance between A and B is $5\sqrt{a}$. Find the value of a.

Q.5 A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60º to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is –

EXERCISE - 4 [PREVIOUS YEARS AIEEE / JEE MAIN QUESTIONS]

Q.1 A ball whose kinetic energy is E, is projected at an angle of 45º to the horizontal. The kinetic energy of the ball at the highest point of its flight will – **[AIEEE 2002]**

(A) E (B) $E\sqrt{2}$
(C) E/2 (D) Zero

Q.2 A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10m from the ground ? **[AIEEE-2003]**

(A) 4.33 m (B) 2.60 m
(C) 8.66 m (D) 5.20 m

Q.3 A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' be the time of flights in the two cases, then the product of the two time of flights is directly proportional to – **[AIEEE-2004]**

(A) $1/R^2$ (B) 1/R
(C) R (D) R^2

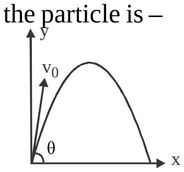
Q.4 A ball is thrown from a point with a speed v_0 at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection θ ? **[AIEEE-2004]**

(A) Yes, 60° (B) Yes, 30°
(C) No (D) Yes, 45°

Q.5 A particle is projected at 60º to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE-2007]**

(A) K (B) zero
(C) K/4 (D) K/2

Q.6 A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is – **[AIEEE 2010]**



(A) $-mgv_0t^2\cos\theta\,\hat{j}$ (B) $mgv_0t\cos\theta\,\hat{k}$

(C) $-\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$ (D) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$

Q.7 A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is – **[AIEEE 2011]**

(A) $\pi\frac{v^2}{g}$ (B) $\pi\frac{v^4}{g^2}$ (C) $\frac{\pi}{2}\frac{v^4}{g^2}$ (D) $\pi\frac{v^2}{g^2}$

Q.8 A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be : **[AIEEE 2012]**

(A) $20\sqrt{2}$m (B) 10 m (C) $10\sqrt{2}$m (D) 20m

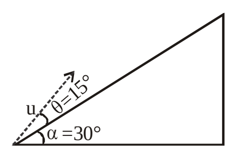
Q.9 A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[JEE MAIN 2013]**

(A) $y = x - 5x^2$ (B) $y = 2x - 5x^2$
(C) $4y = 2x - 5x^2$ (D) $4y = 2x - 25x^2$

Q.10 Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is : **[JEE MAIN 2019 (JAN)]**

(A) 1 : 2 (B) 1 : 4
(C) 1 : 8 (D) 1 : 16

Q.11 A plane is inclined at an angle α = 30° with a respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1} from the base of the plane, making an angle θ = 15° with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to : (Take g = 10 ms^{-2}) **[JEE MAIN 2019 (APRIL)]**



(A) 14 cm (B) 20 cm
(C) 18 cm (D) 26 cm

Q.12 The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then (g = 10 ms^{-2}) : **[JEE MAIN 2019 (APRIL)]**

(A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m/s

(B) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m/s

(C) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m/s

(D) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m/s

Q.13 A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is: **[JEE MAIN 2019 (APRIL)]**
(A) R/g (B) R/4g
(C) 2R/g (D) R/2g

Q.14 Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct? **[JEE MAIN 2019 (APRIL)]**
(A) $R^2 = 2\,h_1h_2$ (B) $R^2 = 16\,h_1h_2$
(C) $R^2 = 4\,h_1h_2$ (D) $R^2 = h_1h_2$

EXERCISE - 5 (PREVIOUS YEARS AIPMT/NEET EXAM QUESTIONS)

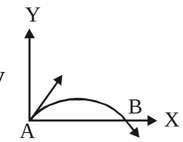
Q.1 The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is – **[AIPMT (MAINS) 2010]**
(A) 60° (B) 15°
(C) 45° (D) 30°

Q.2 A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is – **[AIPMT (PRE) 2011]**
(A) 20 m (B) 40 m
(C) 50 m (D) 60 m

Q.3 A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is : **[AIPMT (MAINS) 2011]**
(A) 60° (B) $\tan^{-1}(1/2)$
(C) $\tan^{-1}(\sqrt{3}/2)$ (D) 45°

Q.4 The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectiles is **[AIPMT (PRE) 2012]**
(A) $\theta = \tan^{-1}(1/4)$ (B) $\theta = \tan^{-1}(4)$
(C) $\theta = \tan^{-1}(2)$ (D) $\theta = 45°$

Q.5 The velocity of a projectile at the initial ...... $(2\hat{i} + 3\hat{j})$ m/s. It's velocity (in m/s) at point B is – **[NEET 2013]**



(A) $2\hat{i} + 3\hat{j}$ (B) $-2\hat{i} - 3\hat{j}$
(C) $-2\hat{i} + 3\hat{j}$ (D) $2\hat{i} - 3\hat{j}$

Q.6 A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3m/s at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is (given g = 9.8 ms^{-2}) **[AIPMT 2014]**
(A) 3.5 (B) 5.9
(C) 16.3 (D) 110.8

Q.7 The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at t = 2s is – **[NEET 2017]**
(A) 5 m/s^2 (B) – 4 m/s^2
(C) – 8 m/s^2 (D) 0

ANSWER KEY

EXERCISE - 1

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	A	C	A	B	B	A	C	A	A	C	A	A	B	C	A	A	A	D	C	C	A	D	C	B
Q	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48		
A	C	C	A	A	D	A	A	C	B	B	C	A	D	C	B	D	C	D	A	B	A	A	B		

EXERCISE - 2

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	C	A	C	C	D	A	A	A	B	A	C	C	C	C	D	D	B	B

EXERCISE - 3

Q	1	2	3	4	5
A	4	10	2	2	5

EXERCISE - 4

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	C	C	C	A	C	C	B	D	B	D	B	A	C	B

EXERCISE - 5

Q	1	2	3	4	5	6	7
A	A	B	B	B	D	A	B

PROJECTILE MOTION TRY IT YOURSELF

(1) (D).

(A) At t_1, the projectile is at the top of its arc. At that point, its velocity vector is comprised of its x-component (that component stays the same throughout the motion as there are no x-direction forces and, hence, nox - d i r e c t i o n acceleration acting to change it) but no y-direction velocity (it's at the top of its arc and, hence, will go no farther upward). That means the velocity direction at the top is in the horizontal. The acceleration vector throughout the motion is in the y-direction (i.e., gravity pointing down), so the velocity vector and the acceleration vectors are perpendicular to one another. This response is true.

(B) At the top, the only velocity component that is non-zero is the horizontal component. As the x-component of the velocity will be constant throughout the motion, and as that component does, indeed, equal $v_o\cos\theta$, this response is true.

(C) The x-component of the acceleration is zero. Twice zero is still zero, so this statement is true.

(2) (B).

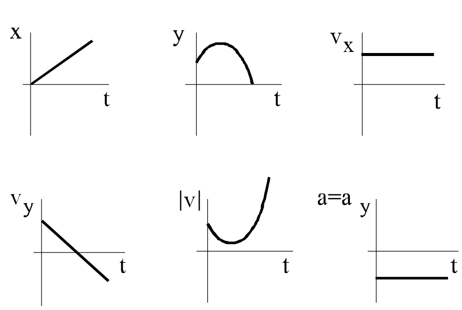
(A) The only acceleration acting is gravity in the y-direction. It is a constant, so this statement is false.

(B) This is the same as saying that the acceleration of the body is a constant, which it is. This statement is true.

(C) The y-component of acceleration is that of gravity. Its sign is negative. The y-component of the velocity going upward is in the direction of motion, or positive. Clearly the two are not the same.

(D) Assuming there is no friction, the velocity magnitude when the body is at h going up will be the same as when going down, but the directions will be different. As we are dealing with vectors, this difference in direction makes the velocities different.

(3) (C). A graph for each of the major parameters for this situation is shown below. This is something you should have been able to both visualize and sketch on your own. If you think you wouldn't have been able to do that, use the graphs provided as a stimulus to do the visualization part.



(4) (D).

(A) Using $x_2 = x_1 + v_1t + 0.5at^2$ with $a = -g$, $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = 0.5\,(-g)t^2$. This selection is evidently true.

(B) The time it takes to hit the ground is a y-motion related question. As the initial velocity in the y-direction for both cases is the same (it's zero), and as the gravitational acceleration is the same in both cases, the two projectiles should hit the ground in the same amount of time. This statement is true.

(C) Because Projectile C had a downward initial velocity in the y-direction, it will take less time to hit the ground than does Projectile D which had no initial velocity in the y-direction. And as Projectile A had an upward y-component of its velocity, it will take more time to reach the ground. This statement is true.

(5) (D).

(A) Using $x_2 = x_1 + v_1t + .5at^2$ with $a = -g$, $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = 0.5\,(-g)\,t^2$. This means that $t = (2h/g)^{1/2}$. If h is doubled, t goes up by a factor of $(2)^{1/2}$, not by a factor of 2. This response is false.

(B) Using $v_2 = v_1 + at$ with v_2 being the velocity just before hitting the ground, $v_1 = 0$, and $a = -g$, we get $v_2 = -gt$. We have already determined that doubling h does not mean t doubles, so this statement is false.

(C) Acceleration in these cases is always constant in both the x and y-direction. False.

(6) The equation of path of a projectile is,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Comparing this equation with the given relation

$$\tan\theta = \sqrt{3} \qquad \therefore\ \theta = 60^o$$

$u\cos\theta = 1$

$$\therefore\ u = 2\text{ m/s},\quad R = \frac{u^2\sin 2\theta}{g} = \frac{4\times\sin 120^\circ}{g} = \frac{2\sqrt{3}}{g}\text{m}\ ;$$

$$H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{4\times(3/4)}{2g} = \frac{3}{2g}\text{m}$$

(7) If the angle of elevation is θ then

$$R = \frac{u^2\sin 2\theta}{g} = \frac{V^2\sin 2\theta}{g}$$

$$\therefore\ \sin 2\theta = \frac{gR}{V^2} \qquad \therefore\ \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$$

(8) **(a)** The maximum height is given by,

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{(28 \sin 30°)^2}{2(9.8)} m = \frac{14 \times 14}{2 \times 9.8} = 10.0m$$

(b) The time taken to return to the same level is

$T_f = (2v_0 \sin\theta_0)/g = (2 \times 28 \times \sin 30°)/9.8$

$= 28/9.8\ s = 2.9\ s$

(c) The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{(v_0^2 \sin 2\theta_0)}{g} = \frac{28 \times 28 \times \sin 60°}{9.8} = 69m$$

(9) $\vec{u} = 10.0\hat{j},\ \vec{a} = 8.0\hat{i} + 2.0\hat{j}, \quad \vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\vec{r} = 10.0t\ \hat{j} + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})\ t^2$$

(a) x co-ordinate $= 4.0\ t^2 = 16$ or $t = 2s$

y co-ordinate $= 10.0 \times 2 + 1.0 \times 2 \times 2 = 24m$

(b) $\vec{v} = \frac{d}{dt}(\vec{r}) = 8t\hat{j} + (10.0 + 2t)\hat{j}$

At $t = 2s$, $\vec{v} = 16\hat{i} + 14\hat{j}$

$$v = \sqrt{16^2 + 14^2} = \sqrt{256 + 196} = \sqrt{452} = 21.26\ ms^{-1}$$

(10) Maximum horizontal range = 100m

$$\therefore \frac{v^2}{g} = 100 \qquad \text{.........(1)}$$

We know that $v(t)^2 - v(0)^2 = 2a\,[x(t) - x(0)]$

Now, $v(t) = 0,\ v(0) = v,\ x(t) - x(0) = h$ (say)

$\therefore 0^2 - v^2 = 2(-g)h$

or $h = \frac{1}{2} \times \frac{v^2}{g}$

or $h = \frac{1}{2} \times 100m = 50m$ [From eq. (1)]

CHAPTER-4 : PROJECTILE MOTION
EXERCISE-1

(1) **(B).** Given, that $y = \sqrt{3}\ x - (1/2)\,x^2$ ....(1)

The above equation is similar to equation of trajectory of the projectiles

$y = \tan\theta\ x - 1/2\ \frac{g}{u^2\cos^2\theta}\ x^2$ ....(2)

Comparing (1) & (2) we get

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^o$ and $1/2 = (1/2)\ \frac{g}{u^2\cos^2\theta}$

$\Rightarrow u^2\cos^2\theta = g \Rightarrow u^2\cos^2 60 = 10$

$\Rightarrow u^2(1/4) = 10 \Rightarrow u^2 = 40 \Rightarrow u = 2\sqrt{10}$ m/s

(2) **(A).** For angle of elevation of 60º, we have maximum height

$h_1 = \frac{u^2\sin^2 60^o}{2g} = \frac{3u^2}{8g}$

For angle of elevation of 30º, we have maximum height

$h_2 = \frac{u^2\sin^2 30^o}{2g} = \frac{u^2}{8g}\ ;\ \frac{h_1}{h_2} = \frac{3}{1}$

(3) **(C).** $v_y = dy/dt = 8 - 10\,t = 8$, when $t = 0$
(at the time of projection.)

$v_x = dx/dt = 6,\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 6^2} = 10$ m/s

(4) **(A).** Horizontal component of velocity

$v_x = u_x = u\cos\theta = 30 \times \cos 30^o = 15\sqrt{3}$ m/s

Vertical component of the velocity

$v_y = u\sin\theta - gt = 30$

$\sin 30^o - 10 \times 1 = 5$ m/s

$v^2 = v_x{}^2 + v_y{}^2 = 700 \Rightarrow u = 10\sqrt{7}$ m/s

(5) **(B).** $T = \frac{2u\sin\theta}{g} \Rightarrow 2 = \frac{2u\sin\theta}{g} \Rightarrow$ $u\sin\theta = g$

$H = \frac{u^2\sin^2\theta}{2g} = \frac{g^2}{2g} = \frac{g}{2} = 5$ m

(6) **(B).** Let u_1 and u_2 be the initial velocities respectively. If h_1 and h_2 are the heights attained by them, then

$h_1 = \frac{u_1^2}{2g}$ and $h_2 = \frac{u_2^2\sin^2\theta}{2g}$ ...(1)

The times of ascent of balls are equal,

we have $t = u_1/g = u_2\sin\theta/g$

$\therefore\ u_1 = u_2\sin\theta$ ... (2)

From eq. (1) $\frac{h_1}{h_2} = \frac{u_1^2}{u_2^2\sin^2\theta}$ ... (3)

From (2) & (3), $\frac{h_1}{h_2} = \frac{1}{1}$

(7) **(A).** $h_1 = \frac{u^2\sin^2\theta}{2g}$ and $h_2 = \frac{u^2\sin^2(90-\theta)}{2g}$

$\therefore\ h_1 + h_2 = u^2/2g\,(\sin^2\theta + \cos^2\theta) = u^2/2g$

$= \frac{98^2}{2\times 10} = 490$

$h_1 - h_2 = 50,\ \therefore\ h_1 = 270$ m and $h_2 = 220$ m

(8) **(C).** Greatest height attained $h = \frac{u^2\sin^2\theta}{2g}$ ... (1)

Horizontal range

$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$ ... (2)

Given that $R = 2h$

$\Rightarrow \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u^2\sin^2\theta}{2g} \Rightarrow \tan\theta = 2$ ... (3)

Hence $\sin\theta = 2/\sqrt{5}$, $\cos\theta = 1/\sqrt{5}$,

$\therefore$ From (2) $R = 4u^2/5g$

(9) **(A).** $R = H\ ;\ \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g}\ ;\ \tan\theta = 4$

(10) **(A).** $R = \frac{u^2\sin 2\theta}{g} = \frac{u^2}{g}\frac{\sqrt{3}}{2} \Rightarrow \frac{u^2}{g} = \frac{2R}{\sqrt{3}}$

When $\theta = 60^o$, $R_2 = \frac{u^2\sin 120^o}{g} = \frac{u^2\cos 30^o}{g}$

$\Rightarrow R_2 = \frac{2R}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = R$

(11) **(C).** $H = \frac{u^2\sin^2\theta}{2g}\ \therefore\ H \propto u^2$.

If initial velocity be doubled then maximum height reached by the projectile will quadrupled.

(12) **(A).** Range $= \frac{u^2\sin 2\theta}{g}$; when $\theta = 90^o$, $R = 0$

i.e. the body will fall at the point of projection after completing one dimensional motion under gravity.

(13) **(A).** Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

(14) **(B).** Range is given by, $R = \frac{u^2\sin 2\theta}{g}$

On moon $g_m = \frac{g}{6}$. Hence $R_m = 6R$

(15) **(C).** Became vertical downward displacement of both (barrel and bullet) will be equal.

(16) **(A).** $x = 36t \therefore v_x = \frac{dx}{dt} = 36\,m/s$

$y = 48t - 4.9t^2 \quad \therefore v_y = 48 - 9.8t$

at t = 0, v_x = 36 and v_y = 48 m/s
So, angle of projection

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) \text{ or } \theta = \sin^{-1}(4/5)$$

(17) **(A).** $R = ut \Rightarrow t = R/u = 12/8$
Now $h = (1/2)\,gt^2 = (1/2) \times 9.8 \times (12/8)^2 = 11$ m

(18) **(A).** Since angle with the horizontal is 45°, therefore vertical height = range
$19.6 = u \times 2$ or $u = 9.8\ ms^{-1}$

$$\left(\because t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2\text{ sec}\right)$$

(19) **(D).** Range = 150 = ut and $h = \frac{15}{100} = \frac{1}{2} \times gt^2$

or $t^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{100} \qquad \therefore t = \frac{\sqrt{3}}{10}$

$\therefore u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3}\ ms^{-1}$

(20) **(C).** The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path.

(21) **(C).** $S = u \times \sqrt{\frac{2h}{g}} \Rightarrow 10 = u\sqrt{2 \times \frac{5}{10}} \Rightarrow u = 10$ m/s

(22) **(A).** For both cases $t = \sqrt{\frac{2h}{g}}$ = constant.

Because vertical downward component of velocity will be zero for both the particles.

(23) **(D).** $t = \frac{2u\sin 30°}{g\cos 30°} = \frac{2(10)\,(1/2)}{10\,(\sqrt{3}/2)} = \frac{2}{\sqrt{3}}$sec

$R = 10\cos 30°\,t - \frac{1}{2}\,g\sin 30°\,t^2$

$$= \frac{10\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3} = 10 - \frac{10}{3} = \frac{20}{3}\text{m}$$

(24) **(C).** Horizontal component of velocity
$u_H = u\cos 60° = u/2$

$\therefore AC = u_H \times t = \frac{ut}{2}$ and AB = AC sec 30°

$$= \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = ut\sqrt{3}$$

(25) **(B).** $H = \frac{u^2\cos^2\beta}{2g}$; $u\cos\beta = \sqrt{2gH}$

time, $t = \frac{u\cos\beta}{g} = \sqrt{\frac{2H}{g}}$

(26) **(C).** Use, horizontal range $H = \frac{u^2}{g}\sin 2\alpha$

At α, 90 – α range remains same.

(27) **(C).** $R_{max} = \frac{u^2}{g} = 100 \Rightarrow u = 10\sqrt{10} = 32\,m/s$

(28) **(A).** The range R is same for the angles of projection θ and (90° – θ).

$$\therefore t_1 = \frac{2u\sin\theta}{g},\quad t_2 = \frac{2u\sin(90° - \theta)}{g}$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{4u^2}{g^2}\,\frac{Rg}{2u^2} = \frac{2}{g}R$$

$\therefore t_1 t_2 \propto R$

(29) **(A).** Maximum height h_{max} = 25 m,
Horizontal range, R = ?, Velocity of projection, v = 40 m/s

We know that $h_{max} = \frac{v^2\sin^2\theta}{2g}$

or $\sin^2\theta = \frac{25 \times 2 \times 9.8}{40 \times 40} = 0.30625$ or $\sin\theta = 0.5534$

or $\theta = \sin^{-1}(0.5534) = 33.6°$

Again, $R = \frac{v^2\sin 2\theta}{g} = \frac{40 \times 40\sin 67.2°}{9.8}$

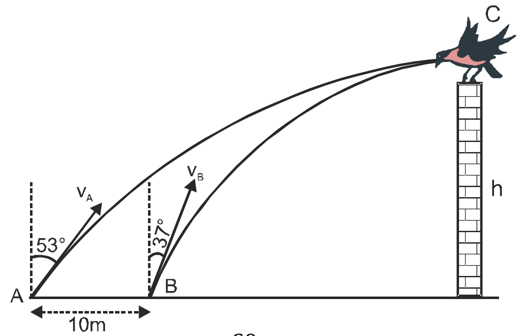
or $R = \frac{1600}{9.8} \times 0.9219\text{m} = 150.5\text{m}$

(30) **(D).** Total time of flight $= \frac{2u\sin\theta}{g} = \frac{2 \times 50 \times 1}{2 \times 10} = 5$ sec
Time to cross the wall = 3 sec (given)
Time in air after crossing the wall = (5 – 3) = 2 sec
∴ Distance travelled beyond the wall

$$= (u\cos\theta)\,t = 50 \times \frac{\sqrt{3}}{2} \times 2 = 86.6\text{ m}$$

(31) **(A).** The vertical components must be equal.

$\therefore v_A\cos 53° = v_B\cos 37°$ or $v_A = v_B\,\frac{\cos 37°}{\cos(90° - 37°)}$



or $v_A = 60 \cot 37° = \dfrac{60}{\tan 37°} = \dfrac{60 \times 4}{3} = 80$ units

(32) **(A).** $V = 2\hat{i} + 4x\hat{j}$, $V = V_x\hat{i} + V_y\hat{j}$

$V_x = 2,\ V_y = 4x\ ;\ \dfrac{dy}{dt} = \dfrac{dy}{dx}.\dfrac{dx}{dt}\ ;\ 4x = \dfrac{dy}{dx}.2$

Integrating, $y = 2\dfrac{x^2}{2} = x^2$

(33) **(C).** $x = 6t,\ \ V_x = \dfrac{dx}{dt} = 6$

$y = 8t - 5t^2,\ \ V_y = \dfrac{dy}{dt} = 8 - 10t$

initial $(t = 0)\ \ V_y = 8$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{6^2 + 8^2} = 10\ m/s$

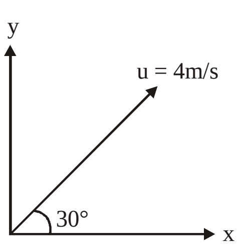
(34) **(B).** $h_1 = \dfrac{u^2 \sin^2 \alpha}{2g}$,

$h_2 = \dfrac{u^2 \sin^2(90 - \alpha)}{2g} = \dfrac{u^2 \cos^2 \alpha}{2g}$

$h_1 h_2 = \dfrac{1}{4}\left(\dfrac{u^2 \sin\alpha \cos\alpha}{g}\right)^2 = \dfrac{1}{4}\left(\dfrac{R}{2}\right)^2$

$R = 4\sqrt{h_1 h_2}$

(35) **(B).**



Components of velocity of ball relative to lift are

$u_x = 4 \cos 30° = 2\sqrt{3}\ m/s$

and $u_y = 4 \sin 30° = 2$ m/s

$T = \dfrac{2u_y}{12} = \dfrac{u_y}{6} = \dfrac{2}{6} = \dfrac{1}{3}s$

(36) **(C).** As, $2 = \dfrac{u^2}{g'}\ \ \therefore\ \ g' = \dfrac{4^2}{2} = \dfrac{36}{2} = 18\ m/s^2$

(37) **(A).** $R = \dfrac{20^2 \times \sin 120°}{g} = 20\sqrt{3} = \dfrac{\Delta R}{R} = \dfrac{24U}{U}$

$\Rightarrow \Delta R = \dfrac{2 \times 5}{100} \times 200\sqrt{3} = 2\sqrt{3}$

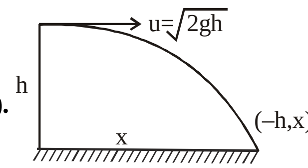
$20\sqrt{3} - 2\sqrt{3} < R < 20\sqrt{3} + 2\sqrt{3}$

$\Rightarrow 31.1 m < R < 38.1\ m$

(38) **(D).** We know that $\dfrac{H}{R} = \dfrac{\tan\theta}{4} \Rightarrow R = \dfrac{4H}{\tan\theta}$

$\Rightarrow R = \dfrac{4 \times 25}{\tan 53°} = \dfrac{4 \times 25}{4/3} = 75\ m$

(39) **(C).**



Using equation to trajectory

$-h = x\tan(0°) - \dfrac{gx^2}{2(2gh)(\cos^2 0°)}$ Þ $x = 2h$

(40) **(B).** PE is maximum at highest point. Hence x = R/2

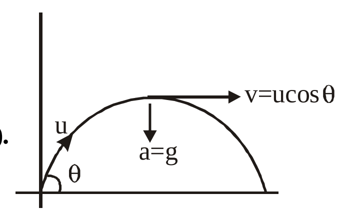
(41) **(D).** Since ranges for angle of projection 2θ and 4θ are same so $2\theta = 90° - 4\theta$

$\Rightarrow\ 6\theta = 90° \Rightarrow \theta = \dfrac{90°}{6} = 15°$

Now, $\dfrac{x}{2} = \dfrac{\sin 2\theta}{\sin 4\theta} = \dfrac{\sin 30°}{\sin 60°} \Rightarrow \dfrac{x}{2} = \dfrac{1/2}{\sqrt{3}/2} \Rightarrow x = \dfrac{2}{\sqrt{3}}$

(42) **(C).** As time in air is same, velocity in y direction should be same hence height should be same.
height and time depends on velocity in y direction)

(43) **(D).**



(44) **(A).** $h = 0 \times t + \dfrac{1}{2}gt^2$ or $t = \sqrt{\dfrac{2h}{g}}$

$S_{jogger} = vt = \sqrt{\dfrac{2hv^2}{g}}$

(45) **(B).** Horizontal velocity $V_x = u_x = 18$ m/s

Now, $\tan 45° = \dfrac{V_y}{V_x}$ $\quad\therefore\ V_y = V_x = 18$ m/s

(46) **(A).** $y = ?, x = 50$

$$y = 50\tan 60° - \frac{g(50)^2}{2(25)^2\cos^2 60°} = 50\sqrt{3} - \frac{9.8\times 4}{2\times(1/4)} = 8.2\text{m}$$

(47) **(A).** As horizontal component of velocity remains constant. Hence, magnitude of change in velocity = magnitude of change in vertical component of velocity

$$= |v_y - u_y| = gt = 10 \times 0.5 = 5 \text{ m/s}$$

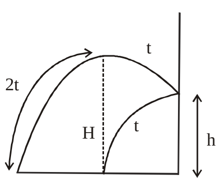
(48) **(B).** $\vec{P}_i = mu\cos 45°\hat{i} + mu\sin 45°\hat{j} = \frac{mu}{\sqrt{2}}\hat{i} + \frac{mu}{\sqrt{2}}\hat{j}$

$$\vec{P}_f = \frac{mu}{\sqrt{2}}\hat{i} - \frac{mu}{\sqrt{2}}\hat{j} \;;\; \vec{P}_f - \vec{P}_i = -\sqrt{2}mu\,\hat{j}$$

EXERCISE-2

(1) **(C).** $H = \frac{1}{2}g(2t)^2 = 2gt^2$ .... (1)

$$h = H - \frac{1}{2}gt^2 \quad \text{.......... (2)}$$



By (1) and (2), $h = H - \frac{H}{4} = \frac{3H}{4}$

(2) **(A).** Take the y axis to be upward and the x axis to the horizontal. Place the origin at the firing point, let the time . . . . θ_0 be the firing angle. If the target is a distance d away, then its coordinate are x = d, y = 0. The kinematic equations are

$$d = v_0 t\cos\theta_0 \text{ and } 0 = v_0 t\sin\theta_0 - \frac{1}{2}gt^2.$$

Eliminate t and solve for θ_0.
The first equation gives $t = d/v_0\cos\theta_0$.
This expression is substituted into the second equation to obtain $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$.
Use the trigonometric identity

$$\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin(2\theta_0) \text{ to obtain } v_0^2\sin(2\theta_0) = gd$$

$$\sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.8\text{ m/s}^2)(45.7\text{m})}{(460\text{ m/s})^2} = 2.12\times 10^{-3}$$

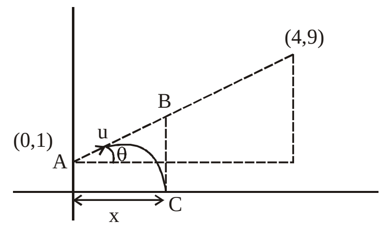
The firing angle is $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \frac{\ell}{d}$

$$\ell = d\tan\theta_0 = (45.7\text{m})\tan 0.0606° = 0.0484\text{m} = 4.84\text{cm}.$$

(3) **(C).** $\tan q = \frac{9-1}{4-0} = 2$ now $-1 = u\sin\theta(1) - \frac{1}{2}g(1)^2$

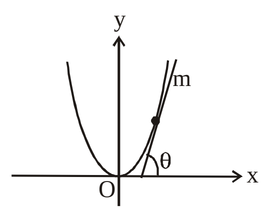
Using $q = 4$ and $\sin q = \frac{2}{\sqrt{5}}$ Þ $u = 2\sqrt{5}$ m/s

now, $x = u\cos q(1) = (2\sqrt{5})$ ´ $\frac{1}{\sqrt{5}} = 2\text{m}$



(4) **(C).** $x^2 = 4ay$

Differentiating w.r.t. y, we get, $\frac{dy}{dx} = \frac{x}{2a}$



$\therefore$ At (2a, a), $\frac{dy}{dx} = 1$, hence $\theta = 45°$

the component of weight along tangential direction is mg sin θ.

Hence tangential acceleration is $g\sin\theta = \frac{g}{\sqrt{2}}$

(5) **(D).** $\theta = 60°$; u = 100 m/s;

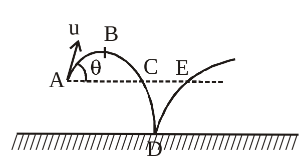
$y = 10^{-3}x^2 = x\sqrt{3} - 5\times 4\times 10^{-4}x^2$

$3\times 10^{-3}\times\sqrt{3} \Rightarrow x = \frac{10^3}{\sqrt{3}}$ m

$y = 10^{-3}\times\frac{10^3}{\sqrt{3}}\times\frac{10^3}{\sqrt{3}} = \frac{10^3}{3} = \frac{1}{3}$ km

(6) **(A).**
(B) Acceleration is upward with horizontal initial velocity so trajectory is parabolic.
(C) Acceleration is zero so velocity is constant.
(D) Due to acceleration speed increase

(7) **(A).** Assume the wall to be absent. Let C and E be two points lying on trajectory at same horizontal level as point of projection.



Then the wall must be placed a distance $d = \frac{AE}{2}$ from A.

The maximum height of ball above ground at B is

$$H = 15 + \frac{10^2}{2 \times g} = 20m$$

$\therefore$ Time taken to fall from B to C is $5 = \frac{1}{2}gt^2$ or $t_1 = 1$ sec.

Time taken to fall from B to D is $t_2 = \sqrt{\frac{2 \times 20}{10}} = 2$ sec.

$\therefore$ Time taken by projectile to move from A to C = 4 sec.
Hence 2d = 4 cos θ × 4 = 40 or d = 20m.

(8) (A), (9) (B), (10) (A).

Slope of V_y versus t graph is – g

$$\therefore -g = \frac{-10}{t_1}$$

As displacement along y-axis is zero $k = -V_y = -10$

$$\tan\alpha = \frac{u_y}{u_x} = 1$$

(11) **(C).** In 2 sec. horizontal distance travelled by bomb $= 20 \times 2 = 40$m.

In 2 sec. vertical distance travelled by bomb

$$= \frac{1}{2} \times 10 \times 2^2 = 20m.$$

In 2 sec. horizontal distance travelled by Hunter $= 10 \times 2 = 20$m.

Time remaining for bomb to hit ground

$$= \sqrt{\frac{2 \times 80}{10}} - 2 = 2 \text{ sec}.$$

Let V_x and V_y be the velocity components of bullet along horizontal and vertical direction.

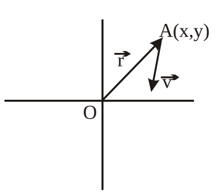
$$\frac{2V_y}{g} = 2 \Rightarrow V_y = 10m/s \text{ and } \frac{20}{V_x - 20} = 2 \Rightarrow V_x = 30 \text{ m/s}$$

Thus velocity of firing is $V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{10}$ m/s

(12) **(C).** If component of velocity along position vector is –ve then distance from origin will be decreasing thus

$$\vec{v}.\vec{r} < 0 \quad \Rightarrow xv_x + yv_y < 0$$

Alternate : OA=distance=$\sqrt{x^2 + y^2}$



If distance is strictly decreasing $\frac{d(OA)}{dt} < 0$

$$\frac{d(OA)}{dt} = \frac{d\sqrt{x^2 + y^2}}{dt} = \frac{1}{2\sqrt{x^2 + y^2}}\left\{2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right\}$$

$$= \frac{1}{2\sqrt{x^2 + y^2}}(2xv_x + 2yv_y)$$

Hence, if $xv_x + yv_y < 0$ then $\frac{d(OA)}{dt} < 0$.

That is OA is decreasing.

(13) **(C), (14) (C), (15) (D).**

x = 2t, y = 5 sin 2t

$v_x = 2$, $v_y = 10 \cos 2t$

$a_x = 0$, $a_y = -20 \sin 2t$

$a = -4y$; $a \propto y$

$v_{max} = \sqrt{14 + 100} = 2\sqrt{26}$

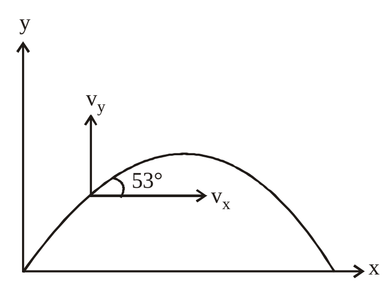
(16) **(D).** Time in case (A) $t = \frac{2u}{g} = 2$sec

(B) : $t = \frac{4 \times 2u}{3g} = \frac{8}{3}$sec (C) : $t = \frac{4 \times 2u \sin 60°}{3g} = \frac{4}{\sqrt{3}}$sec

(D) : $t = \frac{2u}{g + (g/2)} = \frac{4u}{3g} = \frac{4}{3}$sec. Hence min. time in case (D)

(17) **(B).** $\tan 53° = \frac{v_y}{v_x}$

$$\tan 53° = \frac{u\sin\theta - g\left(\frac{T}{2} - 2\right)}{u\cos\theta} \Rightarrow \frac{4}{3} = \frac{u\sin\theta - 10\left(\frac{u\sin\theta}{g} - 2\right)}{u\cos\theta}$$

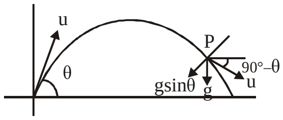


$$\Rightarrow \frac{4}{3} = \frac{u\sin\theta - u\sin\theta + 20}{u\cos\theta} \Rightarrow 4u\cos\theta = 60$$

$$\Rightarrow u\cos\theta = \frac{60}{4} = 15 \Rightarrow u\cos\theta = 15$$

At maximum height vertical component of velocity is zero so velocity at maximum height = u cos θ = 15m/sec

(18)

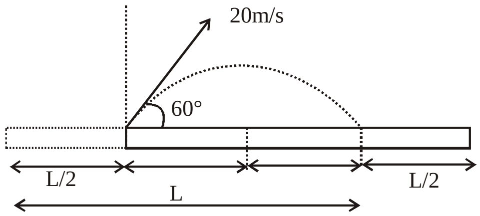


$v\cos(90° - \theta) = u\cos\theta$; $v\sin\theta = u\cos\theta$; $v = u\cot\theta$

At P, $\frac{V_T^2}{R} = a_c$; $\frac{u^2\cot^2\theta}{g\sin\theta} = R$

EXERCISE-3

(1) **4.**



Since the mass is same therefore the length of the plank should be twice the range

$\ell = 2R = 2 \times \frac{u^2 \sin 2\theta}{g} = 40\sqrt{3}$m

(2) **10.** For minimum number of jumps, range must be maximum

Maximum range $= \frac{u^2}{g} = \frac{(\sqrt{10})^2}{10} = 1$ m

Total distance to be covered = 10 meter

So minimum number of jumps = 10

(3) **2.** For horizontal motion of truck and ball

$v\cos 30° \times (t-2) = \frac{1}{2}at^2$

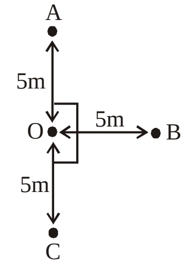
For vertical motion of ball

$\frac{1}{2}g(t-2)^2 = 0$

$\Rightarrow (t-2) = \frac{2v\sin 30°}{g} = \frac{v}{g} \Rightarrow t = 2 + \frac{v}{g}$

$\frac{\sqrt{3}v}{2} \times \frac{v}{g} = \frac{1}{2}a\left(2+\frac{v}{g}\right)^2$; $\frac{\sqrt{3}v^2}{g} = a\left(2+\frac{v}{g}\right)^2$

(4) **2.** Let the stones be projected at t = 0 sec. with a speed u from point O. Then an observer, at rest at t = 0 and having constant acceleration equal to acceleration due to gravity, shall observe the three stones move with constant velocity as shown.



In the given time each ball shall travel a distance 5m as seen by this observer. Hence the required distance between A and B will be $\sqrt{5^2 + 5^2} = 5\sqrt{2}$

(5) **5.** $T = \frac{2u\sin\theta}{g}$

$T = \frac{2 \times 10 \times \sqrt{3}}{10 \times 2} = \sqrt{3}$ sec ; $R = u\cos\theta . T - \frac{1}{2}aT^2$

$1.15 = 10 \times \frac{1}{2}\sqrt{3} - \frac{1}{2}a(\sqrt{3})^2$

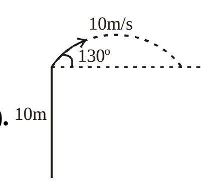
$\frac{3}{2}a = 5\sqrt{3} - 1.15$; $\frac{3a}{2} = 8.65 - 1.15 = 7.5$

$a = 7.5 \times \frac{2}{3} \approx 5\ \text{m/sec}^2$

EXERCISE-4

(1) K.E. at highest point = $E\cos^2\theta = E\cos^2 45° = \frac{E}{2}$

(2) **(C).**



Range $R = \frac{u^2\sin 2\theta}{g} = \frac{(10)^2 \sin 60°}{10} = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$

(3) **(C).** $T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

$T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2u^2\sin 2\theta}{g^2}$; $T_1T_2 = \frac{2R}{g}$

(4) **(A).** Horizontal velocity of ball = $v_0\cos\theta$

$v_0\cos\theta = \frac{v_0}{2}$; $\theta = 60°$

(5) **(C).** Kinetic energy at highest point = $K\cos^2\theta = K/4$

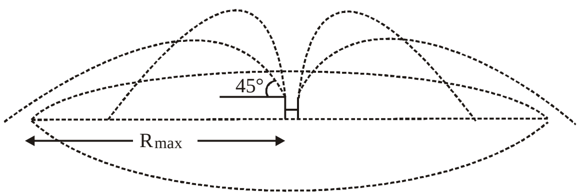
(6) **(C).** $\vec{L} = m(\vec{r}\times\vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\,\hat{j}\right]$$

$$\times\,[v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\,\hat{j}]$$

$$= mv_0\cos\theta\, t - \left[\frac{1}{2}gt\right]\hat{k} = -\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$$

(7) **(B).** $R_{max} = \frac{v^2}{g}\sin 2\theta = \frac{v^2}{g}$; Area $= \pi R^2 = \pi\frac{v^4}{g^2}$



(8) **(D).** $h_{max} = \frac{u^2}{2g} = 10$; $u^2 = 200$ .... (1)

$$R_{max} = \frac{u^2}{g} = 20m$$

(9) **(B).** $\vec{v} = \hat{i} + 2\hat{j}$; $x = t$ .... (1)

$y = 2t - \frac{1}{2}(10t^2)$ .... (2)

From eq. (1) and eq. (2). $y = 2x - 5x^2$

(10) **(D).** $R = \frac{u^2\sin 2\theta}{g}$; $A = \pi R^2$; $A \propto R^2$; $A \propto u^4$

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

(11) **(B).** $t = \frac{2\times 2\times\sin 15°}{g\cos 30°}$; $S = 2\cos 15°\times t - \frac{1}{2}g\sin 30°\,t^2$

Put values and solve, S = 20 cm

(12) **(A).** Equation of trajectory is given as

$y = 2x - 9x^2$ .... (1)

Comparing with equation :

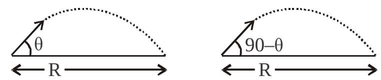
$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$ .... (2)

We get; $\tan\theta = 2 \therefore \cos\theta = \frac{1}{\sqrt{5}}$

Also, $\frac{g}{2u^2\cos^2\theta} = 9$

$\frac{10}{2\times 9\times(1/\sqrt{5})^2} = u^2 \Rightarrow u^2 = \frac{25}{9}$; $u = 5/3$ m/s

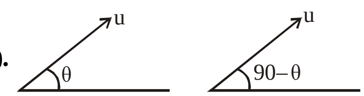
(13) **(C).** Range will be same for time t_1 & t_2, so angles of projection will be θ & $90° - \theta$.



$t_1 = \frac{2u\sin\theta}{g}$, $t_2 = \frac{2u\sin(90°-\theta)}{g}$ and $R = \frac{u^2\sin 2\theta}{g}$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right] = \frac{2R}{g}$$

(14) **(B).**



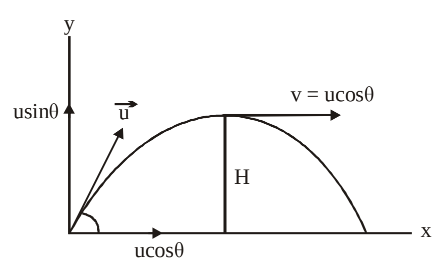
For same range angle of projection will be θ & $90 - \theta$.

$R = \frac{u^2 2\sin\theta\cos\theta}{g}$; $h_1 = \frac{u^2\sin^2\theta}{g}$; $h_2 = \frac{u^2\sin^2(90-\theta)}{g}$

$$\frac{R^2}{h_1h_2} = 16$$

EXERCISE-5

(1) **(A).** Let v be velocity of a projectile at maximum height H



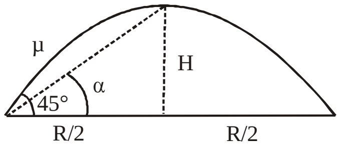
$v = u\cos\theta$

According to given problem, $v = u/2$

$$\therefore \frac{u}{2} = u\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

(2) **(B).** $R_{max} = \frac{u^2}{g} = \frac{(20)^2}{10} = 40m$

(3) **(B).**



$$H = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g} \quad \text{......... (1)}$$

$$R = \frac{u^2 \sin^2 90°}{g} = \frac{u^2}{g}$$

$$\therefore \quad \frac{R}{2} = \frac{u^2}{2g} \quad \text{......... (2)}$$

$$\therefore \quad \tan\alpha = \frac{H}{R/2} = \frac{u^2/4g}{u^2/2g} = \frac{1}{2} \quad \therefore \ \alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

(4) **(B).** Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$ .... (1)

maximum height $H = \frac{u^2 \sin^2\theta}{g}$ ..... (2)

Here, eq. (1) = eq. (2)

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g} \ ; \ 2\cos\theta = \frac{\sin\theta}{2}$$

$\theta = \tan^{-1}(4)$

(5) **(D).** From the figure the X-component remains unchanged, while the Y-component is reverse. Then, the velocity at point B is $2\hat{i} - 3\hat{j}$ m/s.

(6) **(A).** $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

For equal trajectories for same angle of projection

$$\frac{g}{u^2} = \text{constant} \Rightarrow \frac{9.8}{5^2} = \frac{g'}{3^2}$$

$$g' = \frac{9.8 \times 9}{25} = 3.528 \text{ m/s}^2 = 3.5 \text{ m/s}^2$$

(7) **(B).** $v_x = 5 - 4t, \ v_y = 10$

$a_x = -4, \ a_y = 0 \ \vec{a} = a_x\hat{i} + a_y\hat{j} = -4\,\hat{i} \text{ m/s}^2$