Ex: 4.3

17) Find the roots of the following quadratic equations if they exist by the method of completing the square:

i) $2x^2 - 7x + 3 = 0$

Ans) $\frac{2x^2}{2} - \frac{7x}{2} + \frac{3}{2} = 0$

$x^2 - \frac{7x}{2} = \frac{-3}{2}$

$(x)^2 - 2x \cdot \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = \frac{-3}{2}$

$\left(x - \frac{7}{4}\right)^2 = \frac{-3}{2} + \frac{49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{-3 \times 8 + 49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{-24 + 49}{16}$

$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$x - \frac{7}{4} = \sqrt{\frac{25}{16}}$

$x - \frac{7}{4} = \frac{5}{4}$

$x = \frac{5}{4} + \frac{7}{4} = \frac{12}{4} = 3$

(ii) $2x^2 + x - 4 = 0$

Ans) $\frac{2x^2}{2} + \frac{x}{2} - \frac{4^2}{2} = 0$

$x^2 + \frac{x}{2} - 2 = 0$

$x^2 + \frac{x}{2} = 2$

$(x)^2 + 2 \cdot x \cdot \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 2$

$\left(x + \frac{1}{4}\right)^2 = 2 + \frac{1}{16}$

$\left(x + \frac{1}{4}\right)^2 = \frac{2 \times 16 + 1}{16}$

$\left(x + \frac{1}{4}\right)^2 = \frac{32 + 1}{16}$

$\left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$x + \frac{1}{4} = \sqrt{\frac{33}{16}}$

$x + \frac{1}{4} = \frac{\sqrt{33}}{4}$

$x = \frac{\sqrt{33}}{4} - \frac{1}{4} = \frac{\sqrt{33} - 1}{4}$

iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Ans) $\frac{4x^2}{4} + \frac{4\sqrt{3}x}{4} + \frac{3}{4} = 0$

$x^2 + \sqrt{3}x + 3 = 0$

$(x)^2 + 2 \cdot x \cdot \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\left(x - \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{\sqrt{9}}{4}$

$\left(x - \frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} + \frac{3}{4}$

$x - \frac{\sqrt{3}}{2} = 0$

$x = \frac{\sqrt{3}}{2}$

(12) $2x^2+x+4=0$

Ans $\frac{2x^2}{2}+\frac{x}{2}+\frac{4}{2}=0$

$x^2+\frac{x}{2}+2=0$

$x^2+\frac{x}{2}=-2$

$(x)^2+2\cdot x\cdot\frac{1}{4}+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=-2$

$\left(x+\frac{1}{4}\right)^2=-2+\frac{1}{16}$

$\left(x+\frac{1}{4}\right)^2=\frac{-2\times16+1}{16}$

$\left(x+\frac{1}{4}\right)^2=\frac{-32+1}{16}$

$\left(x+\frac{1}{4}\right)^2=\frac{-31}{16}$

$x+\frac{1}{4}=\sqrt{\frac{-31}{16}}$

Hence, square root of negative integer is not possible so the roots do not exist.

Ex - 4.3

2) Find the roots of the quadratic equations given in Q1 above by applying the quadratic formula

i) $2x^2 - 7x + 3 = 0$ $\quad a = 2, b = -7, c = 3$

Sol) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$

$= \frac{7 \pm \sqrt{49 - 4 \times 6}}{4}$

$= \frac{7 \pm \sqrt{49 - 24}}{4}$

$= \frac{7 \pm \sqrt{25}}{4} = \frac{7+5}{4} = \frac{12}{4} = 3$

$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) - \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$

$= \frac{7 - \sqrt{49 - 4 \times 6}}{4}$

$= \frac{7 - \sqrt{49 - 24}}{4} = \frac{7 - \sqrt{25}}{4} = \frac{7-5}{4} =$

$\frac{2}{4} = \frac{1}{2}$

ii) $2x^2+x-4=0 \quad a=2, b=1, c=-4$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$=\frac{(-1)+\sqrt{1^2-4\times2\times(-4)}}{2\times2}$$

$$=\frac{-1+\sqrt{1-8\times(-4)}}{4}$$

$$=\frac{-1+\sqrt{1+32}}{4}$$

$$=\frac{-1+\sqrt{33}}{4}$$

$$x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{(-1)-\sqrt{1^2-4\times2\times(-4)}}{2\times2}$$

$$=\frac{-1-\sqrt{1-8\times(-4)}}{4}$$

$$=\frac{-1-\sqrt{1+32}}{4}$$

$$=\frac{-1-\sqrt{33}}{4}$$

$4x^2 + 4\sqrt{3}x + 3 = 0 \quad a = 4, b = 4\sqrt{3}, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4\sqrt{3} + \sqrt{(4\sqrt{3})^2 - 4 \times 4 \times 3}}{2 \times 4}$$

$$= \frac{-4\sqrt{3} + \sqrt{16\sqrt{9} - 4 \times 12}}{8}$$

$$= \frac{-4\sqrt{3} + \sqrt{16 \times 3 - 48}}{8}$$

$$= \frac{-4\sqrt{3} + \sqrt{48 - 48}}{8}$$

$$= \frac{-4\sqrt{3}}{8}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4\sqrt{3} - \sqrt{(4\sqrt{3})^2 - 4 \times 4 \times 3}}{2 \times 4}$$

$$= \frac{-4\sqrt{3} - \sqrt{16\sqrt{9} - 4 \times 12}}{8} = \frac{-4\sqrt{3} - \sqrt{16 \times 3 - 48}}{8}$$

$$= \frac{-4\sqrt{3} - \sqrt{48 - 48}}{8} = -4\sqrt{3}$$

iv) $2x^2 + x + 4$ $\quad a = 2, b = 1, c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-1) \pm \sqrt{1^2 - 4 \times 2 \times 4}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{1 - 8 \times 4}}{4}$$

$$= \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$= \frac{-1 \pm \sqrt{-31}}{4}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-1) - \sqrt{1^2 - 4 \times 2 \times 4}}{2 \times 2}$$

$$= \frac{-1 - \sqrt{1 - 8 \times 4}}{4} = \frac{-1 - \sqrt{1 - 32}}{4}$$

$$= \frac{-1 - \sqrt{-31}}{4}$$

3) Find the roots of the following equations:

i) $x - \frac{1}{x} = 3, \quad x \neq 0$

$\frac{x^2 - 1}{x} = 3$

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq -4, 7$

$\frac{(x-7) - (x+4)}{(x+4)(x-7)} = -\frac{11}{30}$

$\frac{x - 7 - x - 4}{x(x-7) + 4(x-7)} = \frac{11}{30}$

$\frac{-11}{x^2 - 7x + 4x - 28} = \frac{11}{30}$

$\frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$

$30(-11) = 11(x^2 - 3x - 28)$

$-330 = 11x^2 - 33x - 308$

$11x^2 - 33x - 308 + 330$

$11x^2 - 33x - 36$

$x^2 - 3x - 28 + 30$

$x^2 - 3x + 2 = 0$

$a = 1, \quad b = -3, \quad c = 2$

let roots α and β

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9 - 8}}{2}$$

$$= \frac{3 \pm \sqrt{1}}{2}$$

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) - \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{3 - \sqrt{9 - 8}}{2} = \frac{3 - \sqrt{1}}{2}$$

Q) The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol) Let the present age of Rehman be = x years

Let 3 years ago Rehman's age will be = x - 3 years

After 5 years ago Rehman's age will be = x + 5

ATQ $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\frac{2x+2}{x(x+5)-3(x+5)} = \frac{1}{3}$

$\frac{2x+2}{x^2+5x-3x+15} = \frac{1}{3}$

$\frac{2x+2}{x^2+2x+15} = \frac{1}{3}$

$\frac{3(2x+2)}{x^2+2x+15}$

$\frac{6x+6}{x^2+2x+15}$ $\therefore$ $6x+6 = x^2+2x-15$

$6x+6 = x^2+7x-15$

$x^2+7x-6x-15-6$

$x^2-4x-21=0$

$x^2-7x+3x-21=0$

$x(x-7)+3(x-7)$

$(x-7)(x+7)$

$x=7 \quad x=-3$

The present age of Rehman will be 7 years.

5) In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans) Let the marks of Mathematics be $= x$

Let the marks of English be $= 30-x$

A/Q $(x+2)(30-x-3) = 210$

$(x+2)(27-x) = 210$

$x(27-x) + 2(27-x) = 210$

$27x - x^2 + 54 - 2x = 210$

$27x - 2x - x^2 + 54 - 210 = 0$

$25x - x^2 - 156 = 0$

$-x^2 + 25x - 156 = 0$

$x^2 - 25x + 156 = 0$

$x^2 - 13x - 12x + 156$

$x(x-13) - 12(x-13) = 0$

$(x-12)(x-13)$

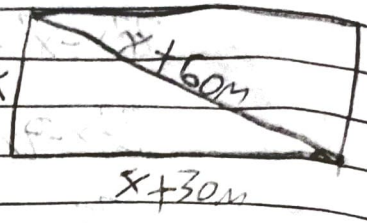
$x = 12 \quad x = 13$

The marks of Mathematics will be $30 - 12 = 18$

The marks of English will be $30 - 13 = 17$

3 | 156
2 | 52
2 | 26
13

6) The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.



Ans) Let the shorter side be x

By phythagoras theorem

$$(x+60)^2 = x^2 + (x+30)^2$$

$$x^2+2\times60x+60^2 = x^2+x^2+60x+30^2$$

$$x^2+120x+3600 = 2x^2+60x+900$$

$$x^2-2x^2+120x-60x+3600-900 = 0$$

$$-x^2+60x+2700 =$$

$$x^2-60x-2700 =$$

$$x^2-90x+30x-2700$$

$$x(x-90)+30(x-90)$$

$$(x+30)(x-90)$$

$x = -30$ $x = 90$ The shorter side will be 90

7) The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans) - Let the larger no be x

Let the smaller no be y

ATQ $x^2 - y^2 = 180$ ... i)

$y^2 = 8x$ ... ii)

Putting the value of y^2 in eq i) we get

$x^2 - 8x - 180 = 0$

$x^2 - 18x + 10x - 180 = 0$

$x(x-18) + 10(x-18) = 0$

$(x+10)(x-18) = 0$

$x = -10 \quad x = 18$

2 | 180
2 | 90
5 | 45
3 | 9
3

The larger no be = 18

Let the smaller side be = $y^2 = 8x18$

$y^2 = 144 \quad y = \sqrt{144} = 12$

Q7 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans) Let the speed of the train be x

Distance covered by the train = 360 km

ATQ $\frac{360}{x} - \frac{360}{x+5} = 1$

$$\frac{360(x+5) - 360x}{x(x+5)} = 1$$

$$\frac{360x + 1800 - 360x}{x^2 + 5x} = 1$$

$$360x + 1800 - 360x = x^2 + 5x$$

$$360x - 360x + 1800 = x^2 + 5x$$

$$1800 = x^2 + 5x$$

$$-x^2 - 5x + 1800$$

$$x^2 + 5x - 1800$$

$x^2+5x-1800$

$x^2+45x-40x-1800$

$x(x+45)-40(x+45)$

$(x-40)(x+45)$

$x=40 \quad x=-45$

2 | 1800
3 | 900
3 | 300
2 | 100
2 | 50
5 | 25
5

The speed of the train will be = 40km/h

9) Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans) Let the tap with smaller diameter can fill the tank alone in x hours.

Let the tap with larger diameter can fill the tank in $= (x-10)$ hours

In 1hr the tap with smaller diameter can fill =

In 1hr the smaller tank is filled = $\frac{1}{x}$

In 1hr the larger tank is filled = $\frac{1}{x-10}$

The tank is filled up in $\frac{3}{8}$ hr $= \frac{75}{8}$

Thus in 1hr the tank is filled up $1 \div \frac{75}{8}$

$= 1 \times \frac{8}{75} = \frac{8}{75}$

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$= \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$= \frac{2x-10}{x^2-10x} = \frac{8}{75}$

$= 75(2x-10) = 8(x^2-10x)$

$= 150x - 750 = 8x^2 - 80x$

$8x^2 - 80x - 150x + 750 = 0$

$= 8x^2 - 80x - 150x + 750 = 0$

$= 8x^2 - 230x + 750 = 0$

$= 4x^2 - 115x + 375 = 0$

$= 4x^2 - 100x - 15x + 375 = 0$

$= 4x(x-25) - 15(x-25) = 0$

$(4x-15)(x-25)$

$x = 25 \quad 4x = 15$

$x = \frac{15}{4}$

Larger tank

$x - 10$

$= 25 - 10 = 15$ hr

(12) An express train takes 1 hour less than a passenger train to travel 132 km between Mysore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Ans: Let the speed of the passenger train be x km/h.

Let the speed of the express train be $(x+11)$ km/h

Time taken by the passenger train to cover 132 km between Mysore to Banglore $= \frac{132}{x}$

Time taken by the express train to cover 132 km between Mysore to Banglore $= \frac{132}{x+11}$

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

$$\frac{132(x+11) - 132x}{x(x+11)} = 1$$

$\frac{137x + 1452 - 33x}{x^2 + 11x} = 1$

$137x - 33x + 1452 = x^2 + 11x$

$1452 = x^2 + 11x$

$= x^2 + 11x - 1452 = 0$

$x^2 + 44x - 33x - 1452 = 0$

$x(x+44) - 33(x+44) = 0$

$(x-33)(x+44) = 0$

$x = 33 \quad x = -44$

The speed of the passenger train = 33 km/h and the speed of express train = 44 km/h

11) Sum of the areas of two squares is 468 m^2. If the difference of their perimeter is 24 m, find the sides of the two squares.

Ans) Let the one side of square be x

Another side be y

Sum of the areas of two squares is $468 m^2$

$x^2 + y^2 = 468 m^2$ ... i)

The difference of their perimeter =

$4x - 4y = 24$ ... ii)

$4x = 24 + 4y$

$x = 6 + y$

$(6+y)^2 + y^2 = 468$

$6^2 + 2 \times 6y + y^2 + y^2 = 468$

$36 + 12y + 2y^2 = 468$

$2y^2 + 12y - 468 + 36 = 0$

$2y^2 + 12y - 432 = 0$

$y^2 + 6y - 216 = 0$

$a = 1, b = 6, c = -216$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (-216)}}{2 \times 1}$

$= \frac{-6 \pm \sqrt{36 + 864}}{2} = \frac{-6 \pm \sqrt{900}}{2} = \frac{-6 \pm 30}{2}$

$$\frac{-6+30}{2} = \frac{24}{2} = 12$$

$$\frac{-6-30}{2} = \frac{-36}{2} = -18$$