Ex 4.4

1) Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

i) $2x^2 - 3x + 5 = 0$

Ans) $D = b^2 - 4ac$

$a = 2, b = -3, c = 5$

$= (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40 = -31$

Nature $D < 0$ No real roots

ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Ans) $D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

Nature $D = 0$ Real and Equal roots

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= x = \frac{-(-4\sqrt{3}) \pm 0}{2 \times 3} = \frac{4\sqrt{3}}{6}$

Roots are $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

iii) $2x^2 - 6x + 3 = 0$

Ans) $D = b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12$

Nature $D > 0$ Distinct real roots

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$

$= \frac{6 \pm \sqrt{4 \times 3}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4}$

$= \frac{2(3 \pm \sqrt{3})}{4}$ or $\frac{3 \pm \sqrt{3}}{2}$

Roots are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2) Find the values of k for each of the following quadratic equations, so that they have two equal roots.

i) $2x^2 + kx + 3 = 0$

$D = b^2 - 4ac \quad a = 2, b = k, c = 3$

$= k^2 - 4 \times 2 \times 3 = 0$

$= k^2 - 24 = 0$

$= k^2 = 24$

$= k = \sqrt{24} = 2\sqrt{6}$

ii) ~~[illegible]~~ $k(x-2)^2 + 6 = 0$

$D = b^2 - 4ac$

$a = k, b = -2k, c = 6$

$= (-2k)^2 - 4 \times k \times 6 = 0$

$= 4k^2 - 24k = 0$

$= 4k^2 = 24k$

$\frac{4k^2}{k} = 24$

$4k = 24$

$k = \frac{24}{4} = 6$

3) Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\ m^2$? If so find its length and breadth.

Ans Let breadth of the rectangular mango grove be x m

Let the length of rectangular mango grove be $2x$ m

ATQ $x \times 2x = 800$ or $2x^2 = 800$

$x^2 = 400$

$x = \pm 20$

Breadth $= 20$ m and

Length $= 2 \times 20 = 40$ m

So, it is possible to design a rectangular mango grove whose length is twice its breadth.

1) Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans) Let the present age of one friend be x years

Then the present age of other friend be $(20-x)$ yrs

4 years ago one friend's age was $(x-4)$ years

4 years ago other friend's age was $(20-x-4)$

$= (16-x)$ years

ATQ $(x-4)(16-x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$

$\Rightarrow x^2 - 20x + 112 = 0$

This is of the form $ax^2 + bx + c = 0$

$a = 1, b = -20, c = 112$

$D = b^2 - 4ac$

$= (-20)^2 - 4 \times 1 \times 112$

$= 400 - 448 = -48$

Nature $D < 0$ No real roots exist. The given equation is not possible.

5) Is it possible to design a rectangular park of perimeter 80m and area $400m^2$? If so, find its length and breadth.

Ans) Let the length of rectangular park be x

Let the breadth of rectangular park be y

Given area $= 400m^2$

$\Rightarrow xy = 400 \Rightarrow y = \frac{400}{x}$

Perimeter $= 80m$

$\Rightarrow 2(x+y) = 80$

$x + y = 40 \quad \ldots\ldots i)$

Putting the value of y in equation i), we get

$x + \frac{400}{x} = 40$

$\Rightarrow x^2 + 400 = 40x$

$\Rightarrow x^2 - 40x + 400 = 0$

$\Rightarrow x^2 - 20x - 20x + 400 = 0$

$\Rightarrow x(x-20) - 20(x-20) = 0$

$\Rightarrow (x-20)(x-20) = 0$

$\Rightarrow x - 20 = 0$ or $x = 20$

Since $y = \frac{400}{x}$

$\Rightarrow y = \frac{400}{20} = 20$

Length of park = 20m and breadth of park = 20m

So, the given situation is possible.