EXERCISE 4.3

(a) Find the roots of the following quadratic equations, if they exist, by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$

Ans - $\frac{2x^2}{2} - \frac{7x}{2} + \frac{3}{2} = 0$

$x^2 - \frac{7x}{2} = -\frac{3}{2}$

$x^2 - \frac{7x}{2} \times \frac{1}{2} = -\frac{3}{2}$

$x^2 - \frac{7x}{4} + \left(\frac{7}{2}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2 = \frac{49}{16}$

$\left[x - \frac{7}{4}\right]^2 = -\frac{24+39}{16}$

$\left[x - \frac{7}{4}\right]^2 = \frac{25}{16}$

$x - \frac{7}{4} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

$x = \frac{5}{4} + \frac{7}{4} = \frac{12}{4} = 3$

$x = \frac{5}{4} - \frac{7}{4} = \frac{-2}{4} = \frac{-1}{2}$

(ii) $2x^2 + x - 4 = 0$

Ans. $\frac{2x^2}{2} + \frac{x}{2} - \frac{4}{2} = 0$

$x^2 + x = 2$

$x^2 + x \times \frac{1}{2} = 2$

$x^2 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 = 2 + \left(\frac{x}{2}\right)^2$

$\left(x + \frac{x}{2}\right)^2 = 2 + \frac{x^2}{4}$

$x + \frac{x}{2} = \sqrt{\frac{8 + x^2}{4}}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Ans. $(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$

$(2x + \sqrt{3})^2 = 0$

$(2x + \sqrt{3}) \cdot (2x + \sqrt{3}) = 0$

$x = \frac{-\sqrt{3}}{2} \quad x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

P.T.O

Ans: $\frac{2}{20}x^2 + \frac{x}{5} + \frac{1}{10} = 0$

$x^2 + x = -2$

$x^2 + \frac{x}{2} = -2$

$x^2 + 2 + \left(\frac{x}{2}\right)^2 = -2 + \left[\frac{x}{2}\right]^2$

$\left[x + \frac{x}{2}\right]^2 = -\frac{8 + x^2}{4}$

$x + \frac{x}{2} = \sqrt{\frac{-8+8}{2}} \quad \sqrt{\frac{-8-8}{2}}$

Q Find the roots of the quadratic equations, if they exist by the method of completing the square:-

(i) $2x^2 - 7x + 3 = 0$

Ans: $a = 2 \quad b = -7 \quad c = 3$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-7 \pm \sqrt{(7)^2 - 4 \times 2 \times 3}}{2 \times 2} = \frac{-7 \pm \sqrt{49 - 24}}{4}$$

$$= \frac{-7 \pm \sqrt{25}}{4}$$

$$\frac{-7 - 5}{4} = \frac{-12}{4} = -3 \quad \frac{-7+5}{4} = \frac{-2}{4} = -\frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

Ans: $a = 2 \quad b = 1 \quad c = -4$

$$= \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4 \times 4 \times -4}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{1 + 32}}{4} = \frac{-1 + \sqrt{33}}{4} \quad \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3} + 3 = 0$

Ans: $a = 4 \quad b = 4\sqrt{3} \quad c = 3$

$$\frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4 \times 4 \times 3}}{2 \times 4} = \frac{-4\sqrt{3} + 0}{8}$$

$$\frac{-\sqrt{3}}{2}$$

$$\frac{-4\sqrt{3} - 0}{8} = \frac{-4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$$

(ii) $2x^2 + x + 4$

Ans: $a = 2 \quad b = 1 \quad c = 4$

$$\frac{-1 \pm \sqrt{(1)^2 + 4 \times 2 \times 4}}{2 \times 2}$$

$$= \frac{-1 + \sqrt{1 + 32}}{4} = \frac{-1 + \sqrt{33}}{4}$$

$$\frac{-1 - \sqrt{33}}{4}$$

4 The sum of the reciprocals of Rehman's age, 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans: Let Rehman age be x

3 years ago rehman age be $x - 3$

P.T.O

5 years : $x+5$

Reciprocal of age : $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{x^2+2x+15} = \frac{1}{3}$$

$$3(2x+2) = 1(x^2+2x+15)$$

$$6x+6 = x^2+2x+15$$

$$x^2+2x-6x-15-6=0$$

$$x^2-4x-21=0$$

$a=1 \quad b=-4 \quad c=-21$

$$= \frac{-b \pm \sqrt{(b)^2-4ac}}{2a}$$

$$\frac{-(-4) \pm \sqrt{(-4)^2-4\times1\times-21}}{2\times1}$$

$$\frac{4+\sqrt{16+84}}{2} \quad \frac{4\pm\sqrt{100}}{2} = \frac{4\pm10}{2} = \frac{14}{2}, \frac{-6}{2}$$

$$\frac{4-10}{2} = -3$$

∴ So, the present age of Rehman is 7 years old

5 In a class test, the sum of Shefali's mark in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans: Let the marks of math be x

mark of English be $30-x$

$(x+2)(30-x-2) = 210$

$(x+2)(27-x) = 210$

$27x + 54 - x^2 - 2x = 210$

$-x^2 + 25x + 54 = 210$

$-x^2 + 25x + 54 - 210 = 0$

$-x^2 + 25x - 156$

$x(x-12) - 13(x-12)$

$(x-12)(x-13)$

$x = 12 \quad x = 13$

6. The diagonal of a rectangle field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans: Let the shorter side be x

Longer side $= x + 30$

Diagonal $= x + 60$

Pythagorean theorem $= (x+60)^2 = (x+30)^2 + x^2$

$x^2 = x^2 + x^2 + 120x - 60x + 3600 - 900$

$-x^2 + 60x + 2700 = 0$

$-(x^2 - 60x - 2700)$

$x^2 - (90-30)x - 2700$

$(x^2 - 90x) + (30x - 2700)$

$x(x-90) + 30(x-90)$

$(x-90)(x+30)$

$x = 90 \quad x = -30$

So, the shorter side is 90m

Longer side $= 90 + 30 = 120$m

Diagonal $= 90 + 60 = 150$m

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans: Larger number be x

Smaller number be $\sqrt{8x}$

$$(x)^2 - (\sqrt{8x})^2 = 180$$

$$x^2 - 8x = 180$$

$$x^2 - 8x - 180 = 0$$

$$a = 1 \quad b = -8 \quad c = -180$$

$$= \frac{-8 \pm \sqrt{(-8)^2 - 4 \times 1 \times -180}}{2 \times 1}$$

$$\frac{8 \pm \sqrt{64 + 720}}{2}$$

$$\frac{8 + \sqrt{784}}{2} = \frac{8 + 28}{2} = \frac{36}{2} = 18$$

$$\frac{8 - 28}{2} = \frac{-20}{2} = -10$$

Larger number = 18

Smaller number = $\sqrt{8 \times 18} = \sqrt{144} = 12$

8. A train travels 360km at uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans: Speed = x

Distance = 360km

Time = $\frac{360}{x}$

$$x+5 = \frac{360}{\frac{360}{x}-1}$$

$$360x = 360 - x + 5$$
$$360x = -x^2 + 360x - 5x + 1800$$
$$360x = -x^2 + 355x + 1800$$
$$-x^2 + 335x - 360x + 1800$$
$$-x^2 - 5x + 1800 = 0$$
$$-(x^2 + 5x - 1800) = 0$$
$$-\frac{5 \mp \sqrt{(5)^2 - 4\times1\times-1800}}{2\times1}$$
$$-\frac{5 \pm \sqrt{25 + 7200}}{2} = -\frac{5 \pm \sqrt{7225}}{2}$$
$$-\frac{5+85}{2}, \frac{80}{2} = 40$$
$$-\frac{5-85}{2}, \frac{-90}{2} = -45$$

∴ So, the speed is 40km/h

9. Two water taps together can fill a tank in $\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans:- Smaller tap be x
Bigger tap be $x-10$

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x-10+x}{x^2-10x} = \frac{8}{75}$$

$\frac{2x-10}{x^2-10x} = \frac{8}{75}$

$75(8x-10) = 8(x^2-10x)$

$150x - 750 = 8x^2 - 80x$

$150x + 80x - 750 - 8x^2 = 0$

$230x - 750 - 8x^2 = 0$

$-8x^2 + 230x - 750 = 0$

$-(8x^2 - 230x + 750) = 0$

$a = 8 \quad b = -230 \quad c = 750$

$\frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$

$\frac{-(-230) \pm \sqrt{(-230)^2 - 4 \times 8 \times 750}}{2 \times 8}$

$\frac{230 \pm \sqrt{52900 - 24000}}{16}$

$\frac{230 \pm 170}{16} = \frac{400}{16} = 25$

$\frac{230 - 170}{16} = \frac{15}{4}$

∴ So, the small tap will take 25 hours the bigger tap will take 25 - 10 15 hours

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Ans:- Passenger Train :- Speed = x

Distance = 132

Time = $\frac{132}{x}$

Express Train: Speed = $x+11$

Distance = 132

Time = $\frac{132}{x+11}$

$$\frac{132}{x+11} - \frac{132}{x}$$

$$\frac{132x - 132x + 1452}{x^2+11x} = \frac{1452}{x^2+11x}$$

$$1452 = x^2 + 11x$$

$$x^2 + 11x - 1452 = 0$$

$$\frac{-11 \pm \sqrt{(11)^2 - 4 \times 1 \times -1452}}{2 \times 1}$$

$$\frac{-11 \pm \sqrt{121 + 5808}}{2}$$

$$\therefore \frac{-11 + \sqrt{5929}}{2} = \frac{-11+77}{2}$$

$$\therefore \frac{66}{2} = 33$$

$$\frac{-11 - 77}{2} = \frac{-88}{2} = -44$$