

① Ampere's circuital law states that line integral of magnetic field around any closed loop is equal to μ_0 times the electric current flowing through the cross-section area enclosed by that loop.

Mathematically, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

Let the current flowing in the solenoid having number of turns per unit length n be I .
 Magnitude of magnetic field inside the solenoid is B while outside is zero.

Now $\oint \mathbf{B} \cdot d\mathbf{l} = \int B \cos 0^\circ \cdot L + \int B \cos 90^\circ \cdot L' + \int B \cos 90^\circ \cdot L + \int B \cos 90^\circ \cdot L'$

The value of first term $\int B \cos 0^\circ \cdot L = BL$.

The second and fourth term are zero because angle between magnetic field and the length loop is 90° .

The third term is also zero as the value of magnetic field outside the solenoid is zero.

Total current flowing through the loop

$I_{\text{total}} = (nL)I$

From Ampere's circuital law, we get $BL = \mu_0 (nL)I$

$\Rightarrow B = \mu_0 nI$



2a) Derivation:- Consider a symmetrical long solenoid having number of turns per unit length equal to n . Let I be the current flowing in the solenoid. then by right hand rule, the magnetic field is parallel to the axis of the solenoid.

Field outside the solenoid: Consider a closed path $abcd$. Applying Ampere's law to this path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

(since net current enclosed by path is zero.)

$$\text{As } dl \neq 0 \therefore B = 0.$$

\therefore Outside the solenoid the magnetic field is zero.

Field Inside the solenoid: Consider a closed path $pqrs$. The line integral of magnetic field vector B along path $pqrs$ is

$$\oint_{pqrs} \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} + \int_{qr} \vec{B} \cdot d\vec{l} + \int_{rs} \vec{B} \cdot d\vec{l} + \int_{sp} \vec{B} \cdot d\vec{l} \quad \text{--- (1)}$$

For path pq , \vec{B} and $d\vec{l}$ are along the same direction.

$$\therefore \int_{pq} \vec{B} \cdot d\vec{l} = \int B dl = Bl$$

For paths qr and sp , \vec{B} and $d\vec{l}$ are mutually perpendicular.

$$\therefore \int_{qr} \vec{B} \cdot d\vec{l} = \int_s \vec{B} \cdot d\vec{l} = \int B \cdot dl \cos 90^\circ = 0$$

For path rs : $B = 0$ (since field is zero outside a solenoid)

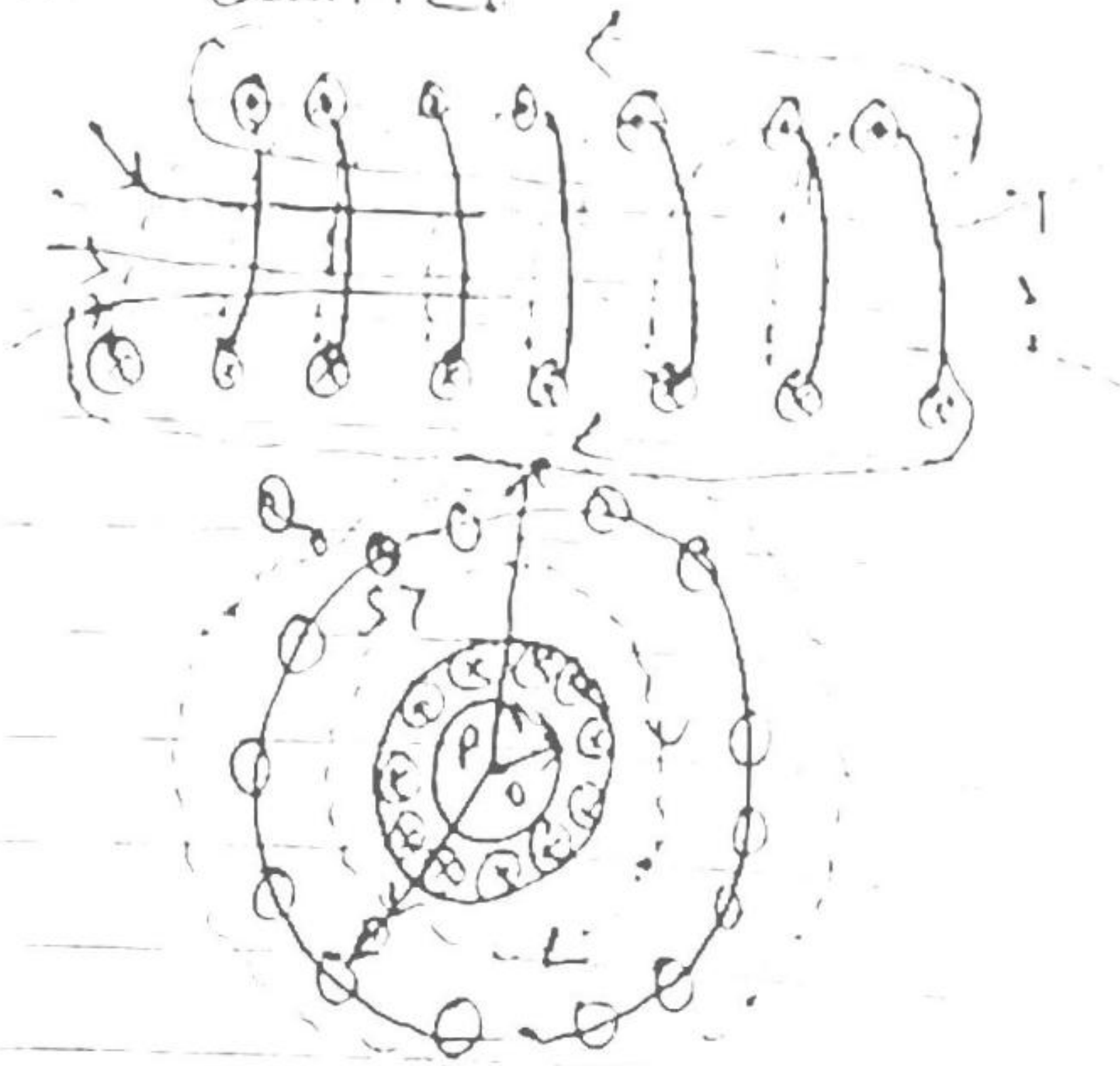
$$\therefore \int_{rs} \vec{B} \cdot d\vec{l} = 0$$

From equation (i) gives

$$\int_{pqrs} \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} = BL = \mu_0 n I l$$

By Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{net current}$
 $\therefore BL = \mu_0 (nI) \therefore B = \mu_0 n I$

b) In a toroid, magnetic lines do not exist outside the body. (toroid is closed whereas solenoid is opened on both sides) magnetic field is uniform inside a toroid whereas for a solenoid, it is different at the two ends and centre.



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2) The magnetic field lines of solenoid are circular having common centre, Inside a given solenoid the magnetic field may be made strong by

(i) passing large current and.

(ii) using laminated coil of soft iron.

(3) Given that

$$n = 300$$

$$I = 5 \text{ A}$$

$$l = 0.5 \text{ m}$$

$$r = 0.01 \text{ cm}$$

$$\frac{l}{r} = \frac{0.5}{0.01 \times 10^{-2}} = 100 \Rightarrow l \gg r$$

$$\begin{aligned} B &= \mu_0 n I = 4\pi \times 10^{-7} \times 300 \times 5 \\ &= 20 \times 3000 \times \pi \times 10^{-7} \\ &= 6000 \times \pi \times 10^{-7} = 6\pi \times 10^{-4} \\ &= 1.88 \times 10^{-3} \text{ T} \end{aligned}$$

(4) Here, $B = 0.52 \times 10^{-3} \text{ T};$
 $\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$
 $l = 0.5 \text{ m}.$
 $N = 500.$

Therefore, number of turns per unit length of the solenoid.

$$n = \frac{N}{l} = \frac{500}{0.5} = 1000 \text{ m}^{-1}$$

If I is the current through the solenoid

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then

$$B = \mu_0 n I$$

$$I = \frac{B}{\mu_0 n} = \frac{2.52 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000} = 2.0 \text{ A}$$