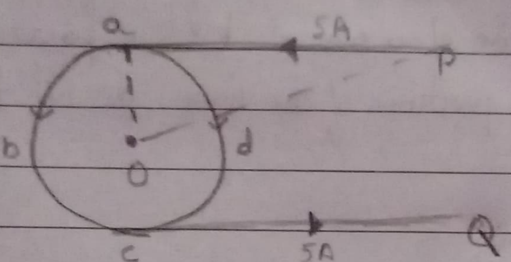
4 July

Date ______
Page ______

H.W.

Q1 In figure abcd is a circular coil of the non-insulated thin uniform conductor. Conductors pa & qc are very long straight parallel conductors tangential to the coil at the ~~pord~~ points a & c. if a current of 5A enters the coil from p to a, find the magnetic induction at O, the center of the coil. The diameter of the coil is 10cm.



Magnetic induction at O due to abc

$$\vec{B_1} = \frac{\mu_0 I}{4r} \text{ (out of the plane)}$$

Magnetic induction at O due to adc

$$\vec{B_2} = \frac{\mu_0 I}{4r} \text{ (into the plane)}$$

$\vec{B_1}$ & $\vec{B_2}$ are equal and opposite directn

So, Mangnetic induction due to coil at O is zero.

$$B_{ap} = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

a - ⊥ distance

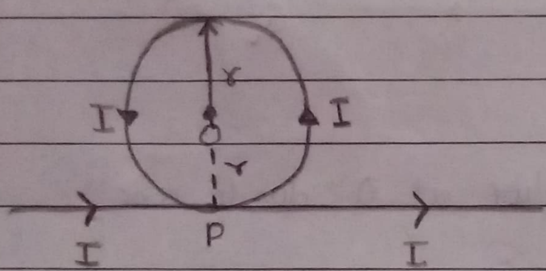
$a = 5 \times 10^{-2}$ cm

$$= 10^{-7} \times \frac{5}{5\times10^{-2}}(\sin 0^\circ + \sin 90^\circ) = 1\times10^{-5}\,T$$

$B_{cq} = B \qquad B_{ap} = 1 \times 10^{-5}\,T$

$B_{net} = 2 \times 10^{-5}\,T$ at the centre O

Q2 A long wire is bent as shown in the figure. What will be the magnitude and direction of the field at the center O of the circular portion if a current I is passed through the wire? Assume that the various portions of the wire do not touch at point P.



$$B_{\substack{\text{at } O \\ \text{due to coil}}} = \frac{\mu_0 I}{2r} \quad (\text{out of the plane})$$

$$B_{\substack{\text{at } O \\ \text{due to Straight} \\ \text{Wire}}} = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2) \quad (\text{out of the plane})$$

$$= \frac{\mu_0 I}{4\pi r}$$

$a = r$

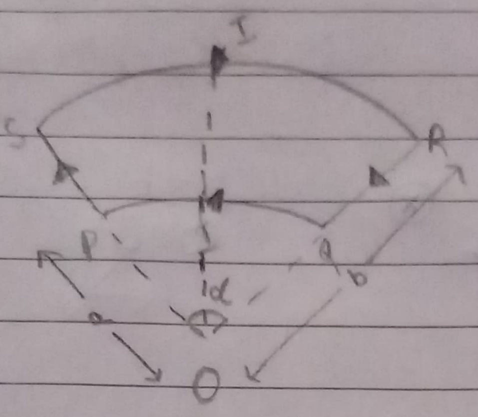
$\theta_1 = \theta_2 = 90^\circ$

(As infinitely straight wire)

$$B_{net} = \frac{\mu_0 I}{2r} + \frac{\mu_0 I}{4\pi r}$$

$$= \frac{\mu_0 I}{2r}\left(1 + \frac{1}{\pi}\right) \odot$$

Q3 Figure shows a current loop having two circular segments and joined by 2 radial lines. Find the magnetic field at the center O.



Magnetic field due to SP & RQ at point O = zero
as angle betn dl & r = $0°$ & $180°$

Magnetic field at O Due to PQ,

$$B = \int dB$$ & $dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\theta}{r^2}$

& $\theta = 90°$

$$= \int \frac{\mu_0}{4\pi}\frac{I dl}{r^2} \sin\theta$$

$$= \int \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl$$ $[l = r\theta]$

$$= \frac{\mu_0}{4\pi}\frac{I}{r^2} r\theta$$

$$= \frac{\mu_0 I \theta}{4\pi r}$$

Now $\theta = \alpha$
$r = a$

B at O Due to PQ $= \frac{\mu_0 I \alpha}{4\pi a}$ ⊙

Magnetic field at O due to SR,

$$= \frac{\mu_0 I \theta}{4\pi r}$$

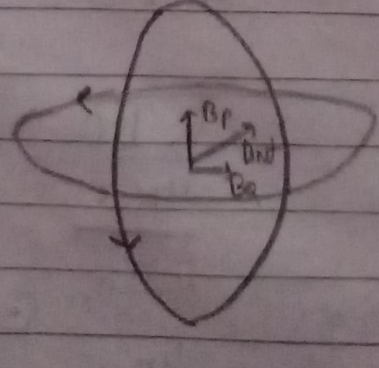
$$\vec{B} = \frac{\mu_0 I \alpha}{4\pi b} \otimes \qquad r = b, \quad \theta = \alpha$$

$$B_{net} = B_1 - B_2$$

$$= \frac{\mu_0 I \alpha}{4\pi a} - \frac{\mu_0 I \alpha}{4\pi b}$$

$$B_{net} = \frac{\mu_0 I \alpha}{4\pi}\left(\frac{b-a}{ab}\right) \odot \quad \text{(Ans)}$$

Q4 Two identical circular coils, P & Q each of radius R, carrying currents 1A & $\sqrt{3}$ A respectively, are placed concentrically & perpendicular to each other lying in the XY & YZ planes. Find the magnitude & direction of the net magnetic field at the centre of the coils.



Magnetic field at the centre of a closed loop $= \frac{\mu_0 I}{2r}$

$$B_{net} = \sqrt{(B_P)^2 + (B_Q)^2}$$

$$= \sqrt{\left(\frac{\mu_0 I_P}{2r}\right)^2 + \left(\frac{\mu_0 I_Q}{2r}\right)^2}$$

$$= \frac{\mu_0}{2R}\sqrt{\cancel{\frac{\mu_0}{2R}}\, I_P^2 + I_Q^2}$$

$$= \frac{\mu_0}{2R}\sqrt{(1)^2 + (\sqrt{3})^2}$$

$$= \frac{\mu_0}{2R}\sqrt{4}$$

$$= \frac{\mu_0}{\cancel{2}R}\times\cancel{2}$$

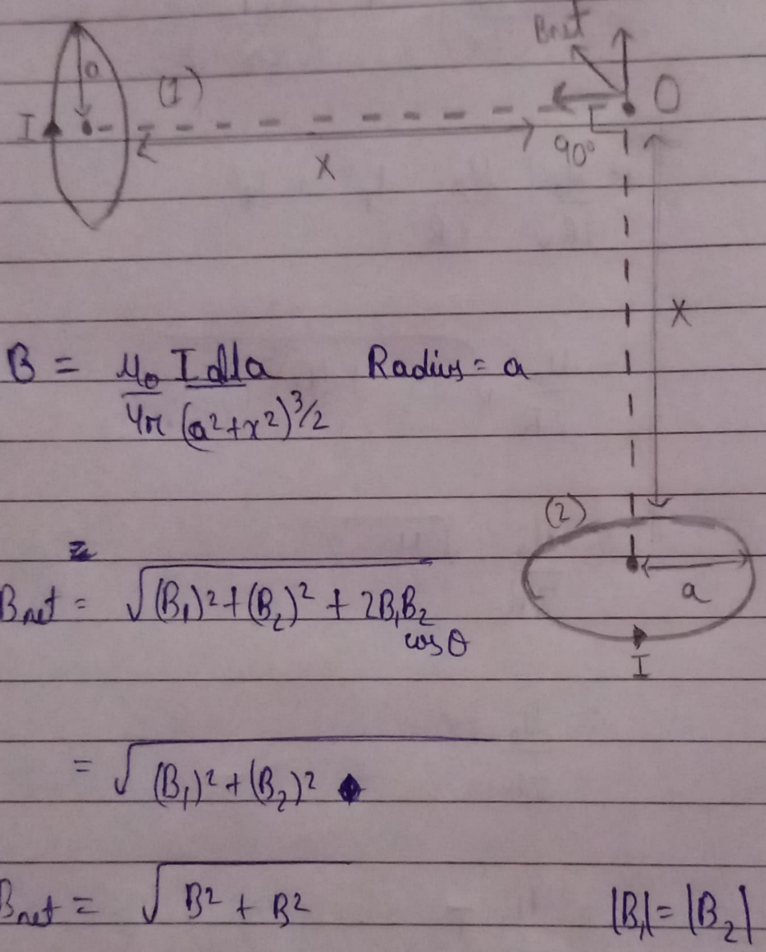
$$= \frac{\mu_0}{R}\,T$$

Resultant magnetic field makes an angle θ with B_Q which is given by

$$\tan\theta = \frac{B_P}{B_Q} = \frac{I_P}{I_Q} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Q. Two very small identical circular loop (1) & (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O



$$B = \frac{\mu_0}{4\pi}\frac{I\,dl\,a}{(a^2+x^2)^{3/2}}$$ Radius = a

$$B_{net} = \sqrt{(B_1)^2+(B_2)^2+2B_1B_2\cos\theta}$$

$$= \sqrt{(B_1)^2+(B_2)^2}$$

$$B_{net} = \sqrt{B^2+B^2}$$ $|B_1| = |B_2|$

$$= \sqrt{2}\,|B|$$

$$= \sqrt{2}\times\frac{\mu_0}{4\pi}\frac{I\,dl\,a}{(a^2+x^2)^{3/2}}$$

$$= \sqrt{2}\times\frac{\mu_0}{2}\frac{I\,a^2}{(a^2+x^2)^{3/2}}$$ $dl = 2\pi a$

$$= \frac{\mu_0 I a^2}{\sqrt{2}\,(a^2+x^2)^{3/2}}$$

Date
Page

Direction of $\hat{B}_2$ = $\hat{j}$, let $\vec{B}_2 = 1\hat{j}$

Direction of $\hat{B}_1$ = $-\hat{i}$, let $\vec{B}_1 = -1\hat{i}$

$\hat{B}_1 + \hat{B}_2$ $\quad \vec{B}_{net} = \vec{B}_1 + \vec{B}_2 \quad \vec{B}_{net} = -\hat{i} + \hat{j}$

$$\hat{B}_{net} = \frac{\vec{B}}{|\vec{B}_{net}|} = \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$$

Direction is along the vector, $\frac{-\hat{i}+\hat{j}}{\sqrt{2}}$