Exercise 7.3

1. ΔABC and ΔDBC are two isosceles triangle on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that.

i) $\Delta ABD \cong \Delta ACD$
ii) $\Delta ABP \cong \Delta ACP$
iii) AP bisects A as well as D.
(iv) AP is the perpendicular bisector of BC.

→ i) ΔABD and ΔACD are similar by SSS congruence because:

$AD = AD$ (Common Arm)
$AB = AC$ (Since ΔABC is isosceles)
$BD = CD$ (Since ΔDBC is isosceles)

$\therefore \Delta ABD \cong \Delta ACD$.

(ii) ΔABP and ΔACP are similar as:
$AP = AP$ (Common side)
$PAB = PAC$ (by CPCT)
$AB = AC$ (Since ΔABC is isosceles)

So, $\Delta ABP \cong \Delta ACP$ by SAS congruence rule.

(ii) $PAB = PAC$ by CPCT as $\Delta ABP \cong \Delta ACP$.
AP bisects A. — (i)

Also, ΔBPD and ΔCPD are similar by SSS congruence congruency as
$PD = PD$ (common side)
$BD = CD$ (since ΔABC is isosceles)
$BP = CP$ (by CPCT as $\Delta ABP \cong \Delta ACP$)

So, $\Delta BPD \cong \Delta CPD$.
Thus, $BDP = CDP$ by CPCT — (ii)

Now by comparing (i) and (ii) it can be said that AP bisects A as well as D.

(iv) $BPD = CPD$ (by CPCT as $\Delta BPD \cong \Delta CPD$) and $BD = CD$ — (i)

$BPD + CPD = 180$ (since BC is a straight line)
$\Rightarrow 2BPD = 180°$
$\Rightarrow BPD = 90°$ — (ii)

Now, from equation (i) and (ii), it can be said that AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that
(i) AD bisects BC
(ii) AD bisects A.

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⇒ (i) In ΔABD and ΔACD,

$ADB = ADC = 90°$

$AB = AC$ (as it is given in the question)

$AD = AD$ (common arm)

$\therefore \Delta ABD \cong \Delta ACD$ by RHS congruence condition

Now, by the rule of CPCT,

$BD = CD$

So, ~~AB~~ AD bisects BC.

(ii) Again, by the rule of CPCT, $BAD = CAD$

Hence, AD bisects A.