Show that the diagonals of a square are equal & bisect each other at right angles.

<u>Given:</u> ABCD ~~is~~ be a square. Diagonals intersect at O.

<u>To prove:</u> we need to prove 3 things.

1. The diagonals of square are equal
2. $OA = OC$ & $OB = OD$,
3. $\angle AOB$, $\angle BOC$, $\angle COD$, $\angle AOD$ is 90 (any one)

<u>Proof</u>

Proof: In $\triangle ABC$ and $\triangle BAD$,

$AB = BA$ (common); $BC = AD$ (opp side of square)

$\angle ABC = \angle BAD$

$\therefore \triangle ABC \cong \triangle BAD$ (SAS rule)

$\therefore AC = BD$ —— (i)

In $\triangle AOD$ and $\triangle BOC$

$AD = CB$ (opp. side of square)

$\angle OAD = \angle OCB$ } Alternate angles are as $AD \parallel BC$, transversal AC

$\angle ODA = \angle OBC$ } transversal BD

$\triangle AOD \cong \triangle BOC$ (ASA); $OA = OC$ and $OB = OD$ (ii) (CPCT)

Now, in $\triangle AOB$ and COB

$AB = BC$ (given); $OA = OC$ (from (ii))

$OB = OB$ (common)

$\triangle AOB \cong COB$ (SSS)

$\angle AOB = \angle BOC$ (CPCT)

But $\angle AOB + \angle BOC = 180°$

$\angle AOB = \angle AOB = 180°$

$2\angle AOB = 180°$

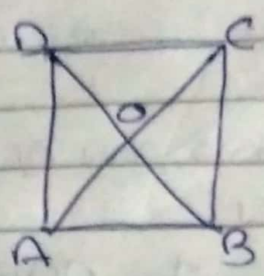
$\angle AOB = 90°$

$\therefore$ AC and BD bisect each at right angles.

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Given: The diagonals AC and BD of a quadrilateral ABCD are equal and bisect each other at right angles.

To prove: Quadrilateral ABCD is a square.



Proof:

In ΔAOD and ΔBOC

$OA = OC$ } Given

$OD = OB$

$\angle AOD = \angle COB$ (V.O.A)

$\therefore \Delta AOD \cong \Delta BOC$ (SAS)

~~AB~~ $AD = BC$; $\angle ODA = \angle OBC$ (CPCT)

$\therefore AD \parallel BC$

Now, $AD = CB$; $AD \parallel CB$

$\therefore$ Quadrilateral ABCD is a $\parallel$gm.

In ΔAOB and ΔAOD,

$AO = AO$ (Common)

$OB = OD$ (Given)

$\angle AOB = \angle AOD$ (Given)

$\Delta AOB \cong \Delta AOD$ (SAS rule)

$AB = AD$

Now, ABCD is a rhombus

Again, in ΔABC and ΔBAD,

$AC = BD$

$BC = AD$; $AB = BA$ (Common)

$\Delta ABC \cong \Delta BAD$ (SSS rule)

$\angle ABC = \angle BAD$ (CPCT)

$\because AD \parallel BC$ (opp. sides of $\parallel$gm ABCD)

$\angle ABC + \angle BAD = 180°$ (Sum of consecutive interior $\angle$s on the same side of transversal is 180°)

$\therefore \angle ABC = \angle BAD$

$= 90°$

Similarly, $\angle BCD = \angle ADC = 90°$

$\therefore$ ABCD is a square