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Exercise 8.2

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid points of sides AB, BC, CD and DA. AC is diagonal. Show that

i) $SR \parallel AC$ and $SR = 1/2\, AC$

ii) $PQ = SR$

iii) PQRS is a parallelogram.

Sol. - Given - ABCD is a quadrilateral in which P, Q, R and S are mid points of AB, BC, CD and DA. AC is a diagonal.

To Prove: i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

ii) $PQ = SR$

iii) PQRS is a parallelogram.

Proof: i) In $\triangle DAC$,

$\because$ S is the mid-point of DA and R is the mid-point of DC

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ (by mid-point of theorem)

(ii) In $\triangle BAC$,

P is the mid-point of AB and Q is the mid-point of BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (by mid-point theorem)

But from i) $SR = \frac{1}{2} AC$ & (ii) $PQ = \frac{1}{2} AC$

$\Rightarrow PQ = SR$

(iii) $PQ \parallel AC$ } (From i)
$SR \parallel AC$ } From (ii)
$\therefore PQ \parallel SR$ (Two lines parallel to the same line are parallel to each other.)

Also, $PQ = SR$ (From (ii))
$\therefore$ PQRS is a parallelogram.
(A quadrilateral is a parallelogram if a pair of opposite sides of parallel and is of equal length)

Q. ABCD is a rhombus and P, Q, R and S are the mid-point of sides AB, CD, BC and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol- Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and ~~AD~~ DA of rhombus.

To prove - PQRS is a rectangle.

Construction: join A and C.

Proof: In $\triangle ABC$, P is the mid of AB and Q is the mid-point of BC.
$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ —— (i)

In $\triangle ADC$, R is the mid-point of CD and S is the mid-point of AD.

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ —— (ii)

From eq. (i) and (ii), PQ ∥ RS and PQ = SR

∴ PQRS is a parallelogram

Now ABCD is rhombus

$AB = BC$

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC$

$\Rightarrow PB = BQ$

$\Rightarrow \angle 1 = \angle 2$ (∠s opp. to equal side are equal)

Now in triangle APS and CQR we have,

$AP = CQ$ (P and Q are the mid-point of AB, BC and AB = BC)

Similarly AS = CR and PS = QR (opposite of side of ∥gm)

∴ $\triangle APS \cong \triangle CQR$ (SSS rule)

$\Rightarrow \angle 3 = \angle 4$ (by C.P.C.T)

Now, we have $\angle 1 + \angle SPQ + \angle 3 = 180°$

$\angle 2 + \angle PQR + \angle 4 = 180°$

$\angle 1 + SPQ + \angle 3 = \angle 2 + PQR + \angle 4$

Since, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (proved above)

∴ $\angle SPQ = \angle PQR$ ——— (iii)

Now, PQRS is a parallelogram (proved above)

∴ $\angle SPQ + \angle PQR = 180°$ ——— (iv)

(co-interior angles)

using eq (iii) and (iv)

$\angle SPQ + \angle SPQ = 180°$

$\Rightarrow 2\angle SPQ = 180°$

$\Rightarrow \angle SPQ = 90°$

PQRS is a rectangle.

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Exercise 8.2

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quad. PQRS is a rhombus.

→ **Given:** A rectangle ABCD in which P, Q, R and S are the mid-point of the sides AB, BC, CD and DA respectively. PQ, QR, SP, RS are joined.

To prove: PQRS is a rhombus

Construction: join AC

Proof: In $\triangle ABC$, P and Q are the mid-points of sides AB, BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ —— (i)

In $\triangle ADC$, R and S are the mid-point of sides CD, AD respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ —— (ii)

From eq. (i) and (ii), $PQ \parallel RS$ and $PQ = SR$ — (iii)

$\therefore$ PQRS is a parallelogram

Now ABCD is a rectangle. (Given)

$AD = BC$

$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$

$\Rightarrow AS = BQ$ —— (iv)

In triangle APS and BPQ,

AP = BP

∠PAS = ∠PBQ

AS = BQ

So △APS and BPQ congruent by SAS congruence

PS = PQ (C.P.C.T) —(V)

From eq. (iv) and (v), we get that PQRS is a ||gm.

⇒ PS = PQ

⇒ Two adjacent sides are equal.

Hence, PQRS is a rhombus

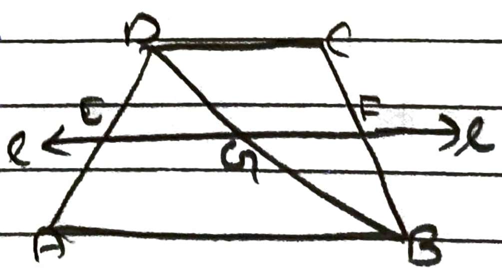
Q. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.

→ Line l || AB and passes through E

line l meets BC in F and BD in G.

In △ABD, E is the mid-point of AD and EG || AB

⇒ G is the mid-point of BD.



Also, l || AB and AB || CD

⇒ l || CD

⇒ F is mid-point of BC (G is the mid-point of BD)

—x—