

Q. A traffic signal board, indicating 'School Ahead', is an equilateral triangle with side 'a'. Find the area of signal boards using Heron's formula. If the perimeter is 180cm, what will the area of the signal board?

$$s = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

$$\begin{aligned} \text{Area of } \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a\right) \left(\frac{3a}{2} - a\right) \left(\frac{3a}{2} - a\right)} \\ &= \sqrt{3 \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}} \\ &= \sqrt{3} \cdot \frac{a}{2} \cdot \frac{a}{2} = \frac{\sqrt{3}}{4} a^2 \end{aligned}$$

(i) 180cm (Perimeter)

$$3 \text{ side} = 180$$

$$\text{Side} = \frac{180}{3} = 60 \text{cm}$$

$$\text{Area of } \Delta = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 60 \times 60 \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} (60)^2 \text{ cm}^2$$

$$= 900\sqrt{3} \text{ cm}^2$$

Heron's formula

Exercise: 12.1

Q2. The triangular side walls of a ~~fraser~~ super tower have been used for advertisements. The side of the walls are 122m, 22m and 120m. The advertisement yield or earning of Rs. 5000 per m^2 per year. A company hired one of its wall for 3 months. How much rent did it pay?

→ Sides of the two triangular walls below the bridge are 122m, 22m and 120m.

$$s = \frac{122m + 22m + 120m}{2} = 132m$$

Area of one triangular wall

$$= \sqrt{132 \times (132 - 122) \times (132 - 22) \times (132 - 120)}$$

$$= \sqrt{132 \times 10 \times 110 \times 12} = 1320m^2$$

Company hired only one wall for 3 months. Thus, earning from advertisements for 3 months at the rate of Rs. 5000 per m^2 per year.

$$\Rightarrow \text{Rs. } 5000 \times \frac{3}{12} \times 1320 = \text{Rs. } 16,50,000$$

Heron's FormulaExercise 12.2

Q2. Find the area of quadrilateral ABCD in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Ans. $a = 4$

$b = 5$

$c = 3$

$$\therefore a^2 + c^2 = b^2$$

$$\Rightarrow \angle B = 90^\circ$$

Area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

For $\triangle ACD$ $a = 4$; $b = 5$ and $c = 5$

$$S = \frac{a+b+c}{2} = \frac{14}{2} = 7 \text{ cm}$$

\therefore Area of $\triangle ACD$

$$= \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{7(7-4)(7-5)(7-5)}$$

$$= \sqrt{7 \times 3 \times 2 \times 2}$$

$$= 2\sqrt{21} \text{ cm}^2$$

Area of quadrilateral ABCD = $6 + 2\sqrt{21} \text{ cm}^2$

$$= 15.2 \text{ cm}^2$$

Q4. ~~A trapezium~~ A triangular and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Ans. Semi perimeter of $\triangle ABC$

$$s = \frac{26+28+30}{2} = 42 \text{ cm}$$

$$s-a = 42-26 = 16 \text{ cm}$$

$$s-b = 42-28 = 14 \text{ cm}$$

$$s-c = 42-30 = 12 \text{ cm}$$

$$\text{Area of } \triangle ABC = \sqrt{(s-a)(s-b)(s-c)}$$

$$= \sqrt{42 \times 15 \times 14 \times 12} \text{ cm}^2$$

$$= \sqrt{14 \times 3 \times 4 \times 4 \times 4 \times 4 \times 3} \text{ cm}^2$$

$$= 14 \times 4 \times 3 \times 2 \text{ cm}^2 = 336 \text{ cm}^2$$

∴ Area of parallelogram = Area of triangle

$$h \times AB = 336$$

$$h \times 28 = 336 \text{ cm}^2$$

$$h = \frac{336}{28} = 12 \text{ cm}$$