

7.4.3

Let the point of intersection of BC and AD be O

$$\begin{aligned} \text{In } \triangle ABO \\ = \angle B < \angle A \end{aligned}$$

Therefore, $AO < BO$
side opposite to greater angle is largest

$$\begin{aligned} \text{Similarly} \\ \text{In } \triangle COD \\ = \angle C < \angle D \end{aligned}$$

$$\text{Therefore } DO < CO$$

side opposite to greater angle is largest

(1) + (2) we get,

$$BO + OC > AO + DO$$

$$= BC > AD$$

Hence proved.

7.4.4

To prove - $\angle A > \angle C$
 $\angle B > \angle D$

In $\triangle ABD$, we see that

$$\angle B < \angle AD < \angle BD$$

So, $\angle AOB < \angle ABD$ — (i) (angle opp. to larger side is larger)

Now, in $\triangle BCD$

$$\angle C < \angle DC < \angle BD$$

Hence, it can be said that

$$\angle BOC < \angle CBD$$
 — (ii)

Now, by adding equation (i) and (ii) we get,

$$\angle AOB < \angle BDC < \angle ABD + \angle CBD$$

$$\angle ADC < \angle ABC$$

$$\boxed{B > D}$$

Similarly in $\triangle ABC$,

$\angle ACB < \angle BAC$ — (iii) (angle opp. to the larger side is always larger)

Now in $\triangle ADC$

$$\angle DCA < \angle DAC$$
 — (iv)

Now by adding equation (iii) and (iv) we

$$\angle ACB + \angle DCA < \angle BAC + \angle DAC$$

$$= \angle BCD < \angle BAD$$

$$\therefore \boxed{A > C}$$

Hence proved.

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Given, $PR > PQ$ is ps bisect $\angle APR$

To prove - $\angle PSR > \angle PSQ$

Proof -

In $\triangle PQR$

As $PR > PQ$

$\therefore \angle PQS > \angle PRS$ - (1) (Angles opp to greater side is greater)

AD, ps bisect $\angle P$ hence

$$\angle QPS = \angle RPS$$
 - (2)

In $\triangle PQS$

$\angle PSR$ is exterior angle

$$\angle QPS + \angle PQS = \angle PSR$$
 - (3)

In $\triangle PRS$

$\angle PSQ$ is exterior angle

$$\angle PRS + \angle RPS = \angle PSQ$$
 - (4)

Adding equation (1) & (2)

$$\angle PQS + \angle QPS > \angle PRS + \angle RPS$$
 - (5)

Put value of equation (3) & (4) in (5)

$$\angle PSR > \angle PSQ$$