Arithmetic Progressions.

Exercise 5.1.

(1)

(i) Taxi fare of each km = ₹15.
Additional fare = ₹8.

$a_1 = 15$

$a_2 = 15 + 8 = 23$

$a_3 = 15 + 2 \times 8 = 15 + 16 = 31$

$a_4 = 15 + 3 \times 8 = 15 + 24 = 39$

AP: 15, 23, 31, 39...

∴ Yes it forms AP as each term is obtained by adding ₹8 to the preceeding term.

(ii) Let the volume be 16 l.
Air removed by pump = $\frac{1}{4} \times 16 = 4$ l.

Air present after removing = 12 - 3 = 9.

AP: 16, 12, 9...

∴ The common difference is not same,

(iii) Cost for first metre = ₹150.
Additional metre = ₹50.

$a_1 = ₹150$

$a_2 = a + d = 150 + 50 = ₹200$

$a_3 = a + 2d = 150 + 2 \times 50 = 150 + 100 = ₹250$

$a_4 = a + 3d = 150 + 3 \times 50 = 150 + 150 = ₹300$

AP: 150, 200, 250, 300.

Yes, because the cost can be obtained by adding the additional cost with the preceeding cost.

(iv) $a_1 = ₹10000 \times \left[1+\frac{8}{100}\right]^2 = 10000 \times \frac{108}{100} = ₹10800$

$a_2 = ₹10000 \times \left[1+\frac{8}{100}\right]^2 = 10000 \times \left[\frac{108}{100}\right]^2 = ₹11664.$

$a_3 = ₹10000 \times \left[1+\frac{8}{100}\right]^2 = 10000 \times \left[\frac{108}{100}\right]^3 = ₹12,597.12$

AP: 10800, 11664, 12597.12.

∴ It does not form an AP.

(2)

(i) $a = 10, d = 10$

$\Rightarrow a_1 = 10$

$\Rightarrow a_2 = a+d = 10+10 = 20$

$\Rightarrow a_3 = a+2d = 10+10\times2 = 10+20 = 30$

$\Rightarrow a_4 = a+3d = 10+10\times3 = 10+30 = 40$

AP: 10, 20, 30, 40.

(ii) $a = -2, d = 0$

$\Rightarrow a_1 = -2.$

$\Rightarrow a_2 = a+d = -2+0 = -2.$

$\Rightarrow a_3 = a+2d = -2+2\times0 = -2.$

$\Rightarrow a_4 = a+3d = -2+3\times0 = -2.$

AP: -2, -2, -2, -2.

(iii) $a = 4, d = -3.$

$\Rightarrow a_1 = 4$

$\Rightarrow a_2 = a+d = 4+(-3) = 4-3 = 1$

$\Rightarrow a_3 = a+2d = 4+2\times-3 = 4-6 = -2$

$\Rightarrow a_4 = a+3d = 4+3\times-3 = 4-9 = -5$

AP: 4, 1, -2, -5

(iv) $a = -1, d = \frac{1}{2}$

$\Rightarrow a_1 = -1$

$\Rightarrow a_2 = a + d = -1 + \frac{1}{2} = \frac{-2+1}{2} = \frac{-1}{2}$.

$\Rightarrow a_3 = a + 2d = -1 + \frac{1}{2} \times 2 = -1 + 1 = 0$.

$\Rightarrow a_4 = a + 3d = -1 + \frac{1}{2} \times 3 = \frac{-1}{2} + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$.

$\therefore$ AP: $-1, \frac{-1}{2}, 0, \frac{1}{2}$.

(v) $a = -1.25, d = -0.25$

$\Rightarrow a_1 = -1.25$

$\Rightarrow a_2 = -1.25 + (-0.25) = -1.25 - 0.25 = -1.5$

$\Rightarrow a_3 = -1.25 + 2 \times (-0.25) = -1.25 - 0.5 = -1.75$

$\Rightarrow a_4 = -1.25 + 3 \times (-0.25) = -1.25 - 0.75 = -2.00$

$\therefore$ AP: $-1.25, -1.5, -1.75, -2.00$

(3)

(i) $3, 1, -1, -3 \cdots$

$\Rightarrow \boxed{a_1 = 3}$.

$\Rightarrow d = 1 - 3$.

$\boxed{d = -2}$

(ii) $-5, -1, 3, 7 \cdots$

$\Rightarrow \boxed{a_1 = -5}$

$\Rightarrow d = -1 - 5$

$\boxed{d = -6}$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \cdots$

$\Rightarrow \boxed{a_1 = \frac{1}{3}}$

$\Rightarrow d = \frac{5}{3} - \frac{1}{3}$

$\boxed{d = \frac{4}{3}}$

(iv) $0.6, 1.7, 2.8, 3.9 \cdots$

$\Rightarrow \boxed{a_1 = 0.6}$.

$d = 1.7 - 0.6$

$\boxed{d = 1.1}$

(iv) $-10, -6, -2, 2$.

$\Rightarrow -6-(-10) = -6+10 = 4$.

$\Rightarrow -2-(-6) = -2+6 = 4$

$\boxed{d = 4}$

$a_5 = a+4d = -10+4\times4 = -10+16 = 6$.

$a_6 = a+5d = -10+4\times5 = -10+20 = 10$.

$a_7 = a+6d = -10+4\times6 = -10+24 = 12$.

$\boxed{\text{AP: } -10, -6, -2, 2, 6, 10, 12}$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}\ldots$

$\Rightarrow 3+\sqrt{2}-3 = \sqrt{2}$.

$\Rightarrow 3+2\sqrt{2}-3+\sqrt{2} = \sqrt{2}$.

$\boxed{d = \sqrt{2}}$

$a_5 = a+4d = 3+4\times\sqrt{2} = 3+4\sqrt{2}$.

$a_6 = a+5d = 3+5\times\sqrt{2} = 3+5\sqrt{2}$.

$a_7 = a+6d = 3+6\times\sqrt{2} = 3+6\sqrt{2}$.

$\boxed{\text{AP: } 3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, 3+4\sqrt{2}, 3+5\sqrt{2}, 3+6\sqrt{2}}$

(vi) $0.2, 0.22, 0.222, 0.2222\ldots$

$\Rightarrow 0.22-0.2 = 0.02$.

$0.222-0.22 = 0.002$

$\boxed{\because \text{It is not an AP}}$

(vii) $0, -4, -8, -12 \cdots$

$\Rightarrow -4 - 0 = -4$

$\Rightarrow -8 - (-4) = -8 + 4 = -4$

$\boxed{d = -4}$.

$a_5 = a + 4d = 0 + 4 \times -4 = -16$.

$a_6 = a + 5d = 0 + 5 \times -4 = -20$

$a_7 = a + 6d = 0 + 6 \times -4 = -24$.

$\boxed{AP: 0, -4, -8, -12, -16, -20, -24}$

(viii) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2} \cdots$

$\Rightarrow \frac{-1}{2} - \left[\frac{-1}{2}\right] = \frac{-1}{2} + \frac{1}{2} = 0$

$\Rightarrow \frac{-1}{2} - \left[\frac{-1}{2}\right] = \frac{-1}{2} - \frac{-1}{2} = 0$

$\boxed{d = 0}$

$a_5 = a + 4d = \frac{-1}{2} + 4 \times 0 = \frac{-1}{2}$.

$a_6 = a + 5d = \frac{-1}{2} + 5 \times 0 = \frac{-1}{2}$.

$a_7 = a + 6d = \frac{-1}{2} + 6 \times 0 = \frac{-1}{2}$.

$\boxed{AP: \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}}$

(iv) 1, 3, 9, 27...

⇒ $3-1=2$.

⇒ $9-3=6$.

It is not an AP.

(v) $a, 2a, 3a, 4a \cdots$

⇒ $2a-a=a$

⇒ $3a-2a=a$.

$d=a$.

$a_5 = a+4d = a+4\times a = a+4a = 5a$.

$a_6 = a+5d = a+5\times a = 6a$.

$a_7 = a+6d = a+6\times a = 7a$.

$\therefore$ AP; $a, 2a, 3a, 4a, 5a, 6a, 7a$

(vi) $a, a^2, a^3, a^4 \cdots$

⇒ $a^2-a = a(a-1)$

⇒ $a^3-a^2 = a^2(a-1)$.

No, it is not an AP

(vii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \cdots$

⇒ $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$.

⇒ $2\sqrt{2}-\sqrt{2}=\sqrt{2}$.

⇒ $3\sqrt{2}-2\sqrt{2}=\sqrt{2}$.

$d=\sqrt{2}$

$a_5 = a+4d = \sqrt{2}+4\times\sqrt{2} = \sqrt{2}+4\sqrt{2} = 5\sqrt{2} = \sqrt{64}$.

$a_6 = a+5d = \sqrt{2}+5\sqrt{2} = \sqrt{2}+5\sqrt{2} = 6\sqrt{2} = \sqrt{128}$

$a_7 = a+6d = \sqrt{2}+6\times\sqrt{2} = \sqrt{2}+6\sqrt{2} = 7\sqrt{2} = \sqrt{256}$

AP; $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \sqrt{64}, \sqrt{128}, \sqrt{256}$

(iii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \cdots$

~~$\sqrt{3}, 2\sqrt{3}, \ldots$~~

$\sqrt{6} - \sqrt{3}$

$\sqrt{3 \times 2} - \sqrt{3}$

$\sqrt{3} \times \sqrt{2} - \sqrt{3}$

$\sqrt{3}(\sqrt{2} - 1)$

$\sqrt{9} - \sqrt{6}$

$\sqrt{3 \times 3} - \sqrt{3 \times 2}$

$\sqrt{3} \times \sqrt{3} - \sqrt{3} \times \sqrt{2}$

$\sqrt{3}(\sqrt{3} - \sqrt{2})$

$\therefore$ It is not an AP.

(iv) $1^2, 3^2, 5^2, 7^2$

$3^2 - 1^2 = 9 - 1 = 8$

$5^2 - 3^2 = 25 - 9 = 16$

Not an AP.

(v) $1^2, 5^2, 7^2, 73 \cdots$

$5^2 - 1^2 = 25 - 1 = 24$

$7^2 - 5^2 = 49 - 25 = 24$

$d = 24$

$a_5 = a + 4d = 1^2 + 4 \times 24 = 1 + 96 = 97$

$a_6 = a + 5d = 1^2 + 5 \times 24 = 1 + 120 = 121$

$a_7 = a + 6d = 1^2 + 6 \times 24 = 1 + 144 = 145$

AP: 1, 25, 49, 73, 97, 121, 145