Exercise 6.4

(1) $\triangle ABC \sim \triangle DEF$

$$\Rightarrow \frac{ar\,\triangle ABC}{ar\,\triangle DEF} = \left[\frac{BC}{EF}\right]^2$$

$$\Rightarrow \frac{64}{121} = \left[\frac{BC}{15.4}\right]^2$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow BC = \frac{\cancel{8}^{4}}{\cancel{11}} \times \frac{\cancel{154}^{14}}{\cancel{10}5}$$

$$= \frac{56}{5}\ cm.$$

$\boxed{BC = 11.2cm}$ Ans

(2) Given:- $AB \parallel DC$

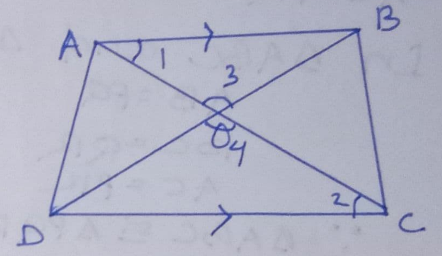
$AB = 2CD$

~~To prove:-~~ In $\triangle AOB$ and $\angle COD$

$\angle 1 = \angle 2$ [Alt- int $\angle$s]

$\angle 3 = \angle 4$ [VOA]

$\triangle AOB \sim \triangle COD$ [By AA]

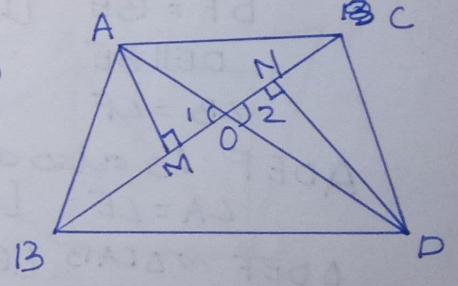


$$\frac{ar\,AOB}{ar\,COD} = \left[\frac{AB}{CD}\right]^2$$

$$\frac{ar\,AOB}{ar\,COD} = \left[\frac{2\cancel{CD}}{\cancel{CD}}\right]^2$$

$$\frac{ar\,AOB}{ar\,COD} = 4 = \boxed{4:1}$$ Ans

(3) Given:- $\triangle ABC$ and $\triangle DBC$ are on same base BC.



To prove:- $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

Construction:- $AM \perp BC$, $BN \perp BC$.

Proof:- In $\triangle AMO$ and $\triangle DNO$,

$\angle 1 = \angle 2$ [VOA]

$\angle M = \angle N$ [90°]

$\therefore \triangle AMO \sim \triangle DNO$ [By AA]

$$\frac{AO}{DO} = \frac{AM}{DN} \quad -(1)$$

$$\frac{ar\,ABC}{ar\,DBC} = \frac{\cancel{\frac{1}{2}} \times \cancel{BC} \times AM}{\cancel{\frac{1}{2}} \times \cancel{BC} \times DN} = \frac{AM}{DN} = \frac{AO}{DO}$$

$$\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$$ [Hence proved]

(4) Let us assume two Δs as $\Delta ABC \sim \Delta PQR$.

$ar(ABC) = ar(PQR)$

$$\Rightarrow \frac{ar(ABC)}{ar(PQR)} = \left[\frac{AB}{PQ}\right]^2 = \left[\frac{BC}{QR}\right]^2 = \left[\frac{CA}{RP}\right]^2.$$

$$\Rightarrow \frac{1}{1} = \left[\frac{AB}{PQ}\right]^2 = \left[\frac{BC}{QR}\right]^2 = \left[\frac{CA}{RP}\right]$$

$$\Rightarrow \left[\frac{AB}{PQ}\right]^2 = \left[\frac{1}{1}\right]$$

$$\Rightarrow \frac{AB}{PQ} = \frac{1}{1}, \quad \frac{BC}{QR} = \frac{1}{1}, \quad \frac{CA}{RP} = \frac{1}{1}$$

$\therefore AB = PQ$

$BC = QR$.

$AC = PR$.

In ΔABC and ΔPQR

$AB = PQ$

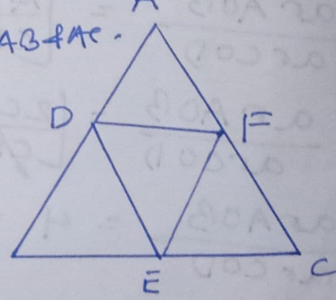
$BC = QR$ [proved above]

$AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ [By SSS].

$\therefore$ Hence proved

(5) D and E are the midpoints of AB & AC.



$DF \parallel BC$

$DF = \frac{1}{2} BC$ [midpoint of BC]

$DF = BE$ [DFBE is a llgm]

$DE \parallel BE$

$\angle B = \angle F$ —(i)

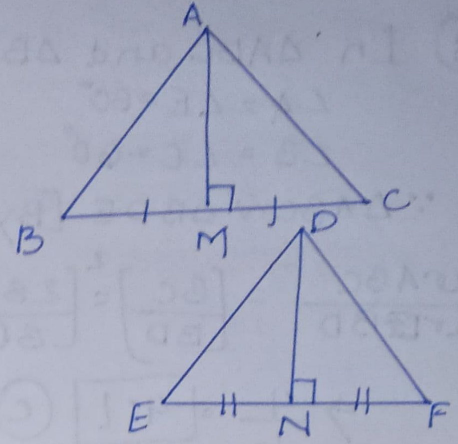
ADEF is also a llgm.

$\angle A = \angle E$ [opp. $\angle s$] —(ii)

$\Delta DEF \sim \Delta CAB$ [By AA]

$$\frac{DEF}{CAB} = \left[\frac{DE}{CB}\right]^2 = \left[\frac{DE}{2DE}\right]^2 = \frac{1}{4} = \boxed{1:4} \text{ Ans}$$

⑥ To prove:- $\left[\frac{AM}{DN}\right]^2 = \frac{ar\,ABC}{ar\,DEF}$



Proof:- $\triangle ABC \sim \triangle DEF$

$\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{AB}{DE} = \frac{\cancel{2}BM}{\cancel{2}EN}$

$\frac{AB}{DE} = \frac{BM}{EN}$

In $\triangle ABM$ and $\triangle DEF$.

$\frac{AB}{DE} = \frac{BM}{EN}$

$\angle B = \angle F$

$\because \triangle ABM \sim \triangle DEF$ [By SAS]

$\frac{AB}{DE} = \frac{AM}{DN}$

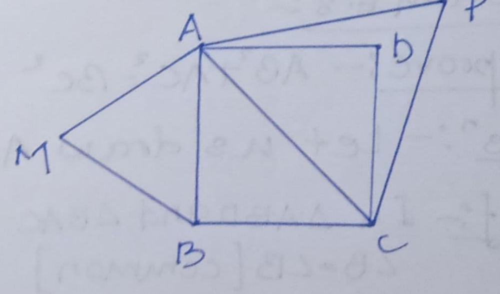
$\left[\frac{AB}{DE}\right]^2 = \left[\frac{AM}{DN}\right]^2.$

$\frac{ar\,ABC}{ar\,DEF} = \left[\frac{AB}{DE}\right]^2 = \left[\frac{AM}{DP}\right]^2$ $\therefore$ Hence proved

⑦ Given:- ABCD is a square

$\triangle ABM$ is an equilateral $\triangle$

$\triangle ACD$ is an equilateral $\triangle$.



To prove:- $\triangle AMB = \frac{1}{2} ar \triangle APC$

Proof:- Let $AB = S$

$AC = \sqrt{S^2 + S^2}$

$= \sqrt{2S^2}$

$= \sqrt{2} \times S$

$\triangle AMB \sim \triangle APC$

$\angle M = \angle P = 60^\circ$

$\angle A = \angle A = 60^\circ$

$\frac{ar\,AMB}{ar\,APC} = \left[\frac{AB}{AC}\right]^2 = \left[\frac{S}{\sqrt{2}S}\right]^2 = \frac{1}{2}.$

$\boxed{\frac{ar\,AMB}{ar\,APC} = \frac{1}{2}.}$ Ans:-

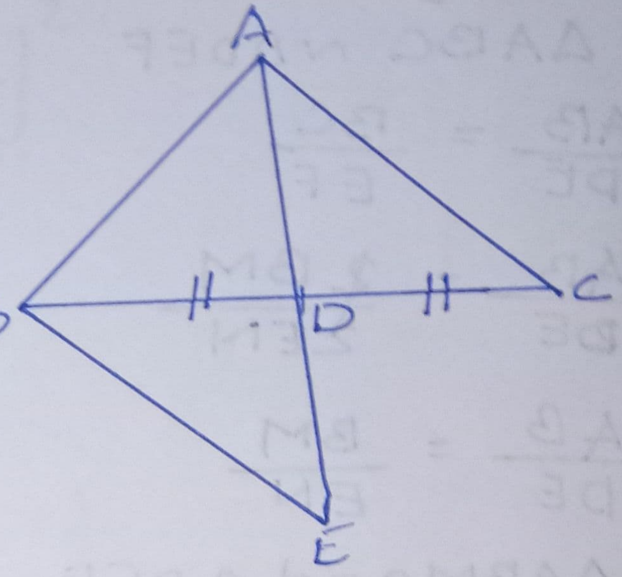
⑧ In ΔABC and ΔBDE

$\angle A = \angle E = 60^\circ$

$\angle B = \angle C = 60^\circ$

$\because \Delta ABC \sim \Delta BDE$ [By AA]

$$\frac{ar\,ABC}{ar\,EBD} = \left[\frac{BC}{BD}\right]^2 = \left[\frac{2BD}{BD}\right]^2 = \frac{4}{1}.$$



$$\Rightarrow \frac{4}{1} = \boxed{4:1}$$ (c)

⑨ Let ΔABC and ΔPQR. be the $\sim \Delta s$.

$$\frac{ar\,ABC}{ar\,PQR} = \left[\frac{AB}{PQ}\right]^2$$

$$\frac{ar\,ABC}{ar\,PQR} = \left[\frac{4}{9}\right]^2.$$

$$\frac{ar\,ABC}{ar\,PQR} = \frac{16}{81}$$

$\boxed{16:81}$ (D)