

Exercise #2

(1) Let $m(2;3)$, $A(-1,7)$ and $B(4,3)$.

$$m = \frac{m_1 m_2 + m_2 m_1}{m_1 + m_2}$$

$$m = \frac{(2 \times 4) + (3 \times -1)}{2 + 3}$$

$$= \frac{8 + (-3)}{5}$$

~~$= \frac{5}{5}$~~

$$= \frac{5}{5} \quad \boxed{m=1}$$

~~$m = 1$~~ $m = (1,3)$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$y = \frac{(2 \times 3) + (-1 \times 7)}{2 + 3}$$

$$y = \frac{-6 + 7}{5}$$

$$y = \frac{1}{5} \quad \boxed{y=3}$$

(2) Let $A(4, -1)$ and $B(-2, -3)$
and, P and Q be the points of trisection.

P divides AB in ratio $1:2$,

$$\begin{aligned}x &= \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \\ &= \frac{(1 \times -2) + (2 \times 4)}{1 + 2} \\ &= \frac{-2 + 8}{3} \\ &= \frac{6}{3} \\ \boxed{x} &= \boxed{2}\end{aligned}$$

$$\begin{aligned}y &= \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \\ &= \frac{(1 \times -3) + (2 \times -1)}{1 + 2} \\ &= \frac{-3 - 2}{3}\end{aligned}$$

$$\boxed{y} = \boxed{\frac{-5}{3}}$$

Q divides AB in ratio $2:1$.

$$\begin{aligned}x &= \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \\ &= \frac{(2 \times -2) + (1 \times 4)}{2 + 1} \\ &= \frac{-4 + 4}{3} \\ &= \frac{0}{3} \\ \boxed{x} &= \boxed{0}\end{aligned}$$

$$\begin{aligned}y &= \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \\ &= \frac{(2 \times -3) + (1 \times -1)}{2 + 1} \\ &= \frac{-6 + (-1)}{3} \\ &= \frac{-6 - 1}{3}\end{aligned}$$

$$\boxed{y} = \boxed{\frac{-7}{3}}$$

$\therefore P\left(2, \frac{-5}{3}\right), Q\left(0, \frac{-7}{3}\right)$

3) D coordinate of Niharika's point $\frac{100}{4} = 25$
on 2nd line
 $(2, 25)$

coordinate of Preet's point $= \frac{100}{5} = 20$
 $(8, 20)$

Let $A(2, 25)$ and $B(8, 20)$

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(8 - 2)^2 + (20 - 25)^2} \\ &= \sqrt{(6)^2 + (-5)^2} \\ &= \sqrt{36 + 25} \\ &= \sqrt{61} \end{aligned}$$

\therefore The distance between two flags is $\sqrt{61}$

Now,

$$\begin{aligned} x &= \frac{x_1 m_2 + x_2 m_1}{m_1 + m_2} \\ &= \frac{8 \times 1 + 2 \times 1}{1 + 1} \\ &= \frac{8 + 2}{2} \\ &= \frac{10}{2} \\ &= 5 \text{ units} \end{aligned}$$

$$\begin{aligned} y &= \frac{y_1 m_2 + y_2 m_1}{m_1 + m_2} \\ &= \frac{20 \times 1 + 25 \times 1}{1 + 1} \\ &= \frac{20 + 25}{2} \\ &= \frac{45}{2} \\ &= 22.5 \text{ units} \end{aligned}$$

$(5, 22.5)$

\therefore She should post her flag on 5th row in 22.5 units.

(a) Let the ratio $m_1:m_2$ be $k:1$
now,

∴

$$\frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = -1$$

$$= \frac{6k + (-3)}{k+1} = -1$$

$$= \frac{6k - 3}{k+1} = -1$$

$$\Rightarrow 6k - 3 = -k - 1$$

$$= 6k + k = -1 + 3$$

$$= 7k = 2$$

$$= \boxed{k = \frac{2}{7}}$$

$$\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 6$$

$$= \frac{8k + 10}{k+1} = 6$$

$$\Rightarrow -8k + 10 = 6k + 6$$

$$\Rightarrow -8k - 6k = 6 - 10$$

$$\Rightarrow -14k = -4$$

$$\Rightarrow k = \frac{-4}{-14}$$

$$= \boxed{k = \frac{2}{7}}$$

∴ The ratio is $2:7$

2 (-4, 5)

The ratio,

$$\frac{5k-5}{k+1}$$

$$\Rightarrow 5k-5 = k+1$$

$$\Rightarrow 5k-k = 1+5$$

$$\Rightarrow 4k = 6$$

(5) Let the ratio be $k:1$.
coordinates of $y = (P, 0)$

there, $m_1 = L, y_1 = -5$
 $m_2 = -4, y_2 = 5$

$$P \left[\frac{k(-4)+1(1)}{k+1}, \frac{k(5)+1(-5)}{k+1} \right]$$

$$= P \left[\frac{-4k+1}{k+1}, \frac{5k-5}{k+1} \right]$$

P lies on m -axis, so its ordinate is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$

$$5k-5 = 0$$

$$5k = 5$$

$$k = 1$$

$$k = 1$$

required ratio = $1:1$.

Now,

$$P \left[\frac{-4k+1}{k+1}, \frac{5k-5}{k+1} \right]$$

$$P \left[\frac{-4(1)+1}{1+1}, \frac{5-5}{2} \right]$$

$$P = \left[\frac{-3}{2}, 0 \right]$$

$$P = \left[\frac{2}{2}, 0 \right]$$

(6)

We know that diagonals of a rhombus intersect each other at their midpoints.

Finding midpoint of AC,

$$\frac{x_1 + x_2}{2} = \frac{y_1 + y_2}{2}$$

$$\frac{1 + 9}{2} = \frac{2 + 6}{2}$$

$$\frac{1 + x}{2} = \frac{8}{2}$$

$$\frac{1 + x}{2} = \frac{8}{2} \quad \text{--- (i)}$$

Finding midpoint of BD,

$$\frac{x_1 + x_2}{2} = \frac{y_1 + y_2}{2}$$

$$\frac{9 + 3}{2} = \frac{y + 5}{2}$$

$$\frac{7}{2} = \frac{y + 5}{2} \quad \text{--- (ii)}$$

Comparing coordinates of x and y.

$$\frac{1 + x}{2} = \frac{8}{2}$$

$$1 + x = 8 \Rightarrow 1 + x = 7$$

$$x = 7 \Rightarrow x = 7 - 1$$

$$\Rightarrow x = 6$$

$$\frac{y + 5}{2} = 4$$

$$y + 5 = 8$$

$$\Rightarrow y = 8 - 5$$

$$\Rightarrow y = 3$$

$\therefore x = 6$ and $y = 3$.

Let the centre be $P(2, -3)$, $B(4)$ and the ratio will be $1:1$.

$$n = \frac{n_1 + n_2}{m_1 + m_2}$$

$$2 = \frac{n_1 + 1}{2}$$

$$\Rightarrow n_1 + 1 = 4$$

$$\boxed{n_1 = 3}$$

$$y = \frac{y_1 + y_2}{m_1 + m_2}$$

$$-3 = \frac{y_1 + 4}{2}$$

$$y_1 + 4 = -6$$

$$\boxed{y_1 = -10}$$

$\therefore A(3, -10)$

$$AP = \frac{3}{7} AB$$

$$\frac{AP}{AB} = \frac{3}{7}$$

$$\frac{AP}{AB} = \frac{7}{3}$$

$$\frac{AP + PB}{AP} = \frac{7}{3} \cdot \frac{3+4}{3}$$

$$\frac{AP}{AP} + \frac{PB}{AP} = \frac{3}{3} + \frac{4}{3}$$

$$1 + \frac{PB}{AP} = \frac{4}{3} + 1$$

$$\frac{PB}{AP} = \frac{4}{3}$$

$$PB : AP = 4 : 3$$

$$n = \frac{m_1 n_2 + m_2 n_1}{m_1 + m_2}$$

$$= \frac{6 + (-8)}{7}$$

$$= \frac{6-8}{7}$$

$$\boxed{n = \frac{-2}{7}}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \frac{-12 + (-8)}{7}$$

$$= \frac{-12-8}{7}$$

$$\boxed{y = \frac{-20}{7}}$$

(10)
(a)

$$\begin{aligned}
 AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-1 - 3)^2 + (4 - 0)^2} \\
 &= \sqrt{(-4)^2 + (4)^2} \\
 &= \sqrt{16 + 16} \\
 &= \sqrt{32}
 \end{aligned}$$

$$\begin{aligned}
 BD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2 - 4)^2 + (-1 - 5)^2} \\
 &= \sqrt{(-6)^2 + (-6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{72}
 \end{aligned}$$

Area of rhombus = $\frac{1}{2} \times \sqrt{32} \times \sqrt{72}$

$$\begin{aligned}
 &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\
 &= \frac{1}{2} \times 48 \\
 &= 24
 \end{aligned}$$

(9) A(-2, 2) and B(2, 8), let the point which divides the line segment be P:

~~$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$~~

$$\begin{aligned}
 x &= \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \\
 &= \frac{2 + (-2)}{2} \\
 &= \frac{2 - 2}{2} \\
 &= \frac{0}{2} = 0
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \\
 &= \frac{8 + 2}{2} \\
 &= \frac{10}{2} \\
 &= 5
 \end{aligned}$$

∴ (0, 5)