Arithmetic Progressions

Exercise 5.1

i) Let a_n be the term form for the first n km.

Then, $a_1 = 15$, $a_2 = 15 + 8 = 23$,

$a_3 = 23 + 8 = 31$

So AP = 15, 23, 31

Hor ~~$a_2 - a_1$~~ $a_3 - a_2 = 8$ $a_2 - a_1 = 8$

So it is an AP

ii) Let the first term be x.

Then $a_1 = x$

$a_2 = x - \frac{1}{4}x = \frac{3}{4}x$

$a_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{9}{16}x$.

$a_4 = \frac{9}{16} + \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{27}{64}x$ units

list = $x, \frac{3}{4}x, \frac{9}{16}x, \frac{27}{64}x, \ldots$

So it is not AP.

iii) First term $a = 150$

common difference $= 200 - 150 = 50$

$a_1 = 150$

$a_2 = 150 + 50 = 200$

$a_3 = 150 + 2 * 50 = 250$

$a_4 = a + 3d = 150 + 150 = 300$

In a $a_2 - a_1 = a_3 - a_2$ so Sorts are AP.

(iv) Let a_n be the amount of money in the n^{th} year

$a_1 = 10000$

$a_2 = 10,000 + 10,000 \times \frac{8}{100}$

$10000 + 800 = 10800$

$a_3 = 10800 + 10800 \times \frac{8}{100}$

$= 10800 + 864 = 11,664$

$a_4 = 11664 + 11,664 \times \frac{8}{100}$

$= 11,664 + 933.12 = 12597.12$

List is 10000, 10800, 11664, 12597.12...

$t_2 - t_1 \neq t_3 - t_2$ So it is not AP

Q2

(i) Given $a = 10, d = 10$

$a_1 = 10$

$a_2 = 10 + 10 = 20$

$a_3 = 20 + 10 = 30$

$a_4 = 30 + 10 = 40$

four terms of AP are 10, 20, 30, 40

(ii) Given $a = -2$, $d = 0$

First four terms of the AP are $-2, -2, -2, -2$.

(iii) $a_1 = 4$, $d = -3$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

First four terms $= 4, 1, -2, -5$

(iv) $a_1 = -1$, $d = \frac{1}{2}$

$a_2 = a_1 + d = \frac{-1}{1} + \frac{1}{2} = \frac{-1}{2}$

$a_3 = a_2 + d = \frac{-1}{2} + \frac{1}{2} = 0$

$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Thus the first four terms of the AP are $-1, \frac{1}{2}, 0, \frac{1}{2}$

(v) $a_1 = -1.25$, $d = -0.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2$

Thus the first four terms are $-1.25, -1.50, -1.75, -2$.

3

(i) $a = 3$ and $d = a_2 - a_1 = 1 - 3 = -2$

(ii) $a = -5$ and $d = a_2 - a_1 = -1 - (-5) = 4$

(iii) $a = \frac{1}{3}$ and $d = a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $a = 0.6$ and $d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

Q.2

(i) 2, 4, 8, 16, ...

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_2 - a_1 \neq a_3 - a_2$

Thus the given sequence is not an AP

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = \frac{3}{1} = \frac{5}{2} = \frac{1}{2}$

$a_2 - a_1 = a_3 - a_2$

Thus the given sequence is an AP.

$a_1 = 2, d = \frac{1}{2}$

Next term $= a_5 = a + 4d$

$= \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}$

$a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = 5$

(iii) $-1.2, -3.2, -5.2, -7.2, \ldots$

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$

$a_3 - a_2 = a_2 - a_1$

Thus, the given sequence is an AP

$a_1 + 1.2d = -2$

$a_5 + a_4 + d = -7.2 + (-2) = -9.2$

$a_6 = a_5 + d = (-9.2) + (-2) = -11.2$

$a_7 = a_6 + d = (-11.2) + (-2) = -13.2$

(iv) $-10, -6, -2, -2, \ldots$

$a_2 - a_1 = -6 - (-10) = 4$

$a_3 - a_2 = -2 - (-6) = 4$

So the sequence is an AP

$a_1 = -10, d = 4$

$a_5 = a_4 + d = 2 + 4 = 6, \quad a_6 = a_5 + d = 6 + 4 = 10$

$a_7 = a_1 + d = 10 + 4 = 14$

(v) $3, 3, -\sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

The sequence is an AP.

Three terms - $a_1 = 3, d = \sqrt{2}$

$a_5 = a_4 + d = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = a_5 + d = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = a_6 + d = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222 ...

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_3 - a_2 \neq a_2 - a_1$

(vii) 0, -4, -8, -12 ...

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -4$

$a_3 - a_2 = a_2 - a_1$

The sequence is an AP.

$a_1 = 0, d = -4$

$a_5 = a_4 + d = -12 + (-4) = -16$

$a_6 = a_5 + d = -16 - 4 = -20$

$a_7 = a_6 + d = -20 - 4 = -24$

(viii) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}$

$a_2 - a_1 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = \frac{-1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = 0$

$a_3 - a_2 = a_2 - a_1$

Sequence is an AP

$a_1 = \frac{-1}{2}, d = 0$

$a_5 = a_4 + d = \frac{-1}{2}$

$a_6 = a_5 + d = \frac{-1}{2}$

$a_7 = a_6 + d = \frac{-1}{2}$

(iv) 1, 3, 9, 27, ...

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_3 - a_2 \neq a_2 - a_1$

Sequence is not an AP.