Exercise 5.3

1.

(a) 2, 7, 12, ... to 10 terms

$a = 2$ $\quad d = 7 - 2 = 5$ $\quad n = 10$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{10}{2}\{2(2) + (10-1)5\}$$

$$= 5\{4 + 45\}$$

$$= 5(49)$$

$$= 245$$

(b) $-37, -33, -29, \ldots$ to 12 terms

$a = -37$ $\quad d = -33 - (-37) = -33 + 37 = 4$

$n = 12$

$$S_n = \frac{12}{2}\{2(-37) + (12-1)4\}$$

$$= 6\{(-74) + 44\}$$

$$= 6\{-74 + 44\}$$

$$= 6(-30)$$

$$= -180$$

0.6, 1.7, 2.8, ... to 100 terms.

$a = 0.6$ $\quad d = 1.7 - 0.6 = 1.1$

$n = 100$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{100}{2}\{2(0.6) + (100-1)1.1$$

$$= 50\{1.2 + 108.9\}$$

$$= 50\{110.1\}$$

$$= 5(1101)$$

~~5505~~

$= 5505$

(iv) $a = \frac{1}{15}$

$n = 11$

$$d = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$$

$$S_{11} = \frac{11}{2}\left\{\frac{2}{15} + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}$$

2

(a) $a = 7, \quad d = \frac{21}{2} - \frac{7}{1} = \frac{7}{2}$

$a_n = 84$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$84 - 7 = (n-1)\frac{7}{2}$

$\frac{7}{7} \times 2 = n-1$

$\Rightarrow 22 = n-1$

$\Rightarrow 23 = n$

$S_n = \frac{n}{2}[a+l]$

$S_{23} = \frac{23}{2}[7+84]$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2}$

$= 1046.5$

(c) $34 + 32 + 30 + \ldots + 0$

$a = 34 \quad d = 32 - 34 = -2 \quad a_n = 10$

$a_n = a + (n-1)d$

$\Rightarrow 10 = 34 + (n-1)(-2)$

$10 - 34 = (n-1)(-2)$

$\frac{-24}{-2} = (n-1)$

$12 = (n-1)$

$12 = n-1$

$13 = n$

$S_{13} = \frac{13}{2}[34+10]$

$= \frac{13}{2} \times 44$

$= 13 \times 22 = 286$

(ii) $-5 + (-8) + (-11) + \ldots\ldots + (-230)$

$a = -5, \quad a_n = -230$

$d = -8 - (-5) = -3$

$a_n = a + (n-1)d$

$a_n = (-5) + (n-1)-3$

$-230 = (-5) + (n-1)(-3)$

$-230 + 5 = (n-1)(-3)$

$-225 = n$

$\frac{-225}{-3} = n-1$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{221}{2}\{2(-8) + (-221-1)-3\}$

$= 75 + 1 = n$

$= 76$

$$S_{76} = \frac{76}{2}[-5 + (-234)]$$

$$= 38 \times (-235)$$

$$= -8930$$

7.

a) $a = 5, \quad d = 3 \quad a_n = 50$ find n and S_n

$$a_n = a + (n-1)d$$

$$\Rightarrow 50 = 5 + (n-1)(3)$$

$$\Rightarrow 50 - 5 = (n-1)(3)$$

$$\Rightarrow \frac{45}{3} = 15 = n - 1$$

$$\Rightarrow 15 + 1 = n$$

$$\Rightarrow 16 = n$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{16}{2}\{2(5) + (16-1)3\}$$

$$= 8\{10 + 45\}$$

$$= 8(55)$$

$$= 440$$

(ii) given $a = 7$, $a_{13} = 35$ find d and s_{13}

$a + 12d = 35$

$7 + 12d = 35$

$12d = 28$

$d = \frac{28}{12} \; 7$

$d = \frac{7}{3}$

$a_n = a + (n-1)d$

$a_{13} = 7 + (13-1)d$

$35 = 7 + 12d$

$35 - 7 = 12d$

$\frac{28}{12} \; 7 = d$

$\frac{7}{3} = d$

$S_{13} = \frac{13}{2} \{ 2(7) + (13-1)\frac{7}{3} \}$

$= \frac{13}{2} \{ 14 + {}^{4}\cancel{12} \times \frac{7}{\cancel{3}} \}$

$= \frac{13}{2} \{ 14 + 28 \}$

$= \frac{13}{2} \{ {}^{21}\cancel{42} \}$

$S_{13} = 273$

(iii) $a_{12} = 37$ $d = 3$ find a and S_{12}

$a_{12} = a + (12-1)3$

$37 = a + 33$

$\Rightarrow 37 - 33 = a$

$\Rightarrow \underline{4 = a}$

$a_{10} = a + 9d$
$= 4 + 9(3)$
$= 4 + 27$
$= 31$

$S_{12} = \frac{12}{2}\{2(4) + (12-1)3\}$
$= 6\{8 + 33\}$
$= 6\{41\}$
$= \underline{246}$

(iv) $a_3 = 15$, $S_{10} = 125$ find d and a_{10}

$a_3 = a + (3-1)d$
$15 = a + 2d$ ——— (i)

$\because S_{10} = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 125 = 5[2a + 9d]$

$\Rightarrow 25 = 2a + 9d$ ——— (ii)

(ii) − (i) $2a + 9d = 25$
(i) × 2 $2a + 4d = 30$
$5d = -5$
$d = \frac{-5}{5}$
$d = 1$

$a + 2d = 15$

$\Rightarrow a + 2(-1) = 15$

$\Rightarrow a + (-2) = 15$

$\Rightarrow a = 15 + 2$

$\Rightarrow a = 17$

$a_{10} = a + 9d$

$= 17 + 9(-1)$

$a_{10} = 17 + (-9)$

$\Rightarrow a_{10} = 17 - 9$

$\Rightarrow a_{10} = 8$

(iv) given $d = 5$ $S_9 = 75$ find a and a_9

$S_9 = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 75 = \frac{9}{2}\{2a + 8(5)\}$

$\Rightarrow 75 = \frac{9}{2}(2a + 40)$

$\Rightarrow 75 = \frac{9}{2} \times 2(a + 20)$

$\Rightarrow 75 = 9a + 180$

$\Rightarrow 75 - 180 = 9a$

$\Rightarrow \frac{-105}{3} = a$

$\Rightarrow -\frac{35}{3} = a$

$a_9 = a + 8d$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35 + (40)3}{3}$

$= \frac{-35 + 120}{3}$

$= \frac{85}{3}$

(vii) a = 2, d = 8, S_n = 90 find n and a_n

Sol: $S_n = \frac{n}{2}(2a + (n-1)d)$

$90 = \frac{n}{2}\{2(2) + (n-1)8\}$

$\Rightarrow 90 = \frac{n}{2}(4 + 8n - 8)$

$\Rightarrow 90 = \frac{n}{2} 4^2(1 + 2n - 2)$

$\Rightarrow 90 = 2n(1 + 2n - 2)$

$\Rightarrow \frac{90}{2n-1} = 2n$

$= \frac{90}{2(n-1)} = 45 = 2n$

$\Rightarrow 90 = 2n + 24n^2 - 4n$

$\Rightarrow 4n^2 - 2n = 90$

$\Rightarrow 4n^2 - 2n - 90 = 0$

$\therefore 2n^2 - n - 45 = 0$

$2n(n-5) + 9(n-5) = 0$

$(n-5)(2n+9) = 0$

$n = 5$ or $n = -\frac{9}{2}$

Sum of the number of terms cannot be negative. So $n = -\frac{9}{2}$ is rejected.

$n = 5$

$a_5 = 2 + 4 \times 8 = 34$

(vi) $a_n = 4$ $d = 2$ $S_n = -14$ find n and a

$4 =$

(vii) given $a = 8$, $a_n = 62$ $S_n = 210$ find n and d.

$a_n = a + (n-1)d$

$62 = 8 + (n-1)d$

$S_n = \frac{n}{2}\{a + l\}$

$\Rightarrow 210 = \frac{n}{2}(8 + 62)$

$\Rightarrow 210 = \frac{n}{2}(70)$

$\Rightarrow \frac{210}{35} = n$

$\Rightarrow 6 = n$

$a_n = 8 + (6-1)d$

$\Rightarrow 62 = 8 + 5d$

$\Rightarrow 62 - 8 = 5d$

$\Rightarrow 54 = 5d$

$\Rightarrow \frac{54}{5} = d$

Q10 given $a_n = 4$ $d = 2$ $S_n = -14$ find n and a

$a_n = a + (n-1)d = 4$

$\Rightarrow a + (n-1)2 = 4$

$\Rightarrow a + 2n - 2 = 4$

$a + 2n = 6$

$\Rightarrow a = 6 - 2n$ ——— (1)

Now $S_n = -14$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = -14$

$\Rightarrow \frac{n}{2}[2(6-2n) + (n-1)2] = -14$

$\Rightarrow \frac{n}{2}[10 - 2n) = -14$

$\Rightarrow n(n-5) = 14$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow (n-7)(n+2) = 0$

$\Rightarrow n = 7, \quad n = -2$

Since $n \neq -ve$

putting $n = 7$ in eq (1)

$a = 6 - 2 \times 7 = -8$

So $n = 7$ and $a = -8$

(ix) Given $a = 3$, $n = 8$ and $S = 192$

$S_8 = 192$

$\Rightarrow \frac{8}{2}(2 \times 3 + 7d) = 192$

$\Rightarrow 4 \times (6 + 7d) = 192$

$\Rightarrow 6 + 7d = \frac{192}{4} = 48$

$\Rightarrow 7d = 42 \Rightarrow d = 6$

(x) given $l = 28$, $S = 144$, $n = 9$

$a_9 = 28 \quad S_9 = 144$

$S_9 = \frac{9}{2}(a + a_n)$

$\Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow a + 28 = \frac{288}{9} = 32$

$\Rightarrow a = 32 - 28$

$\Rightarrow a = 4$