From eq. (i) and eq (2) proved that

$\frac{AM}{AB} = \frac{AN}{AD}$ {by b}

4

In ΔBAC $DE \parallel AC$

$\therefore \frac{BE}{BC} = \frac{BD}{AD}$ —— (i) {by basic proportional theorem}

Similarly in ΔBAE, $DF \parallel AE$

$\therefore \frac{BF}{FE} = \frac{BD}{DA}$ —— (ii) [By proportionality Theorem]

From equations (i) and (ii) we get

$\frac{BC}{BC} = \frac{BF}{FE}$ Hence proved

5

In ΔPOQ

$DE \parallel OQ$

$\frac{PE}{EQ} = \frac{PD}{DO}$

In ΔROR,

$DF \parallel OR$

$\frac{PF}{FR} = \frac{PD}{DO}$

From equation (i) and (ii), we get

$\frac{PE}{EQ} = \frac{PF}{FR}$

$EF \parallel QR$

6 $AB \parallel PQ$

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (i) By Basic proportionality Theorem.

and $AC \parallel PR$

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (ii)

From equations (i) and (ii), we get:

$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore BC \parallel QR$

[By converse of Proportionality Theorem]

and ~~B~~ $AC \parallel PR$

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ ...(ii)

From equations (i) and (ii), we get:

$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore BC \parallel QR$

[By convers of Basic proportionality Theorem]

7

Given: A $\triangle ABC$ in which D is the mid-point of AB and $DE \parallel BC$

To Prove: $AE = EC$

Proof: In $\triangle ABC$, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

But: $AD = DB$

$$\Rightarrow \frac{AD}{DB} = 1$$

$$\Rightarrow 1 = \frac{AE}{EC} \Rightarrow AE = EC$$

Hence, DE bisects AC.

8

The given figure shows a $\triangle ABC$ in which D and E are mid points of sides AB and AC respectively

$$\therefore \frac{AD}{DB} = 1$$

and $\frac{AE}{EC} = 1$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} = 1$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

[by converse of proportionality Theorem]

9. Given: ABCD is a trapezium in which $AB \parallel DC$

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Draw $EO \parallel DC$

Proof in ABD

$EO \parallel DC$ [By const]
$DC \parallel AB$ [Given]
$EO \parallel AB$

$\frac{AE}{ED} = \frac{BO}{DO}$

In $\triangle ADC$, $EO \parallel DC$ = $\frac{AE}{ED} = \frac{AO}{CO}$

From eq i and ii

$\frac{BO}{DO} = \frac{AO}{CO}$ or $\frac{AO}{BO} = \frac{CO}{DO}$

10. $\frac{AO}{BO} = \frac{CO}{OD}$ [Given]

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$. (i)

In $\triangle DAB$, $EO \parallel AB$ [By construction]

$\therefore \frac{DE}{EA} = \frac{DO}{OB}$ [By Basic proportionality Theorem]

$\Rightarrow \dfrac{AE}{ED} = \dfrac{BO}{OD}$ ...(ii)

From equations (i) and (ii), we get:

$\dfrac{AO}{OC} = \dfrac{AE}{ED}$

$\therefore$ $OE \parallel CD$

[By converse of ~~Basic~~ B.P.T]

But we have $AB \parallel OE$

$\therefore$ $AB \parallel CD$

Hence, quadrilateral ABCD is a trapezium

proved.