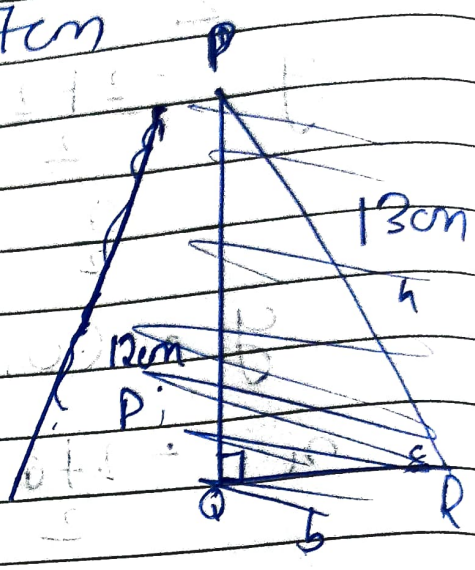
Exercise 8.1

1. $AB = 24\text{cm}$, $BC = 7\text{cm}$

(i) $\sin A$, $\cos A$

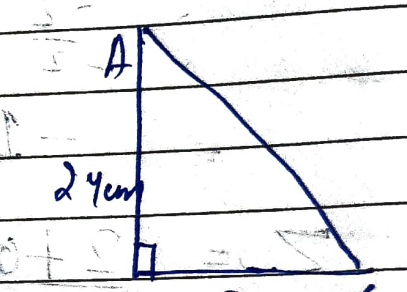
$\sin A = \frac{BC}{AC} = \frac{7}{25}$

$\cos A = \frac{AB}{AC} = \frac{24}{25}$



(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$

$\cos C = \frac{BC}{AC} = \frac{7}{25}$



2.

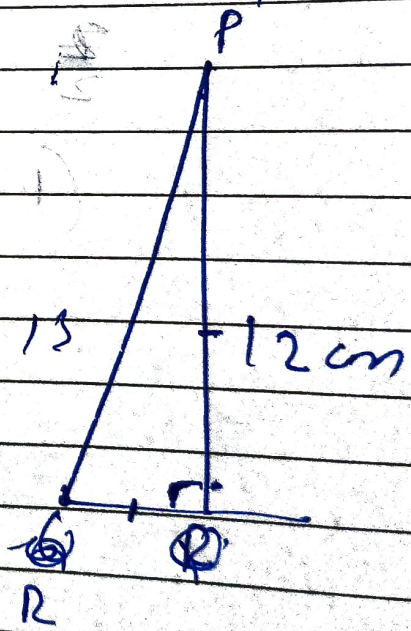
$PR^2 - PQ^2 = QR^2$

$\Rightarrow (13)^2 - (12)^2 = QR^2$

$\Rightarrow 169 - 144 = QR^2$

$\Rightarrow (25)^2 = QR^2$

$\Rightarrow QR = 5$



$\tan P = \frac{5}{12}$

$\cot R = \frac{5}{12}$

$\tan P = \cot R$

$= \frac{5}{12} - \frac{5}{12} = 0$

3. $\sin A = \frac{3}{4}$

$\cos A = ?$

$\tan A = ?$

$\sin A = \frac{P}{h} = \frac{3}{4}$

$P = 3 \quad h = 4$

$b^2 = h^2 - P^2$

$b^2 = (4)^2 - (3)^2$

$b^2 = 16 - 9$

$b^2 = 7$

$b = \sqrt{7}$

$\cos A = \frac{\sqrt{7}}{4}$

$\tan A = \frac{3}{\sqrt{7}}$

4. $15 \cot A = 8$ find $\sin A$ and $\sec A$

$\cot A = \frac{8}{15}$

$\sin A = ?$

$\sec A = ?$

$\cot\theta = \frac{b}{P} = \frac{8}{15} \quad b = 8 \quad P = 15$

$h^2 = P^2 + b^2$

$h^2 = (15)^2 + (8)^2$

$= 225 + 64 = 289$

$h = \sqrt{289}$

$h = 17$

$b = 8 \qquad P = 15 \qquad h = 17$

$\sin A = \frac{P}{h} = \frac{15}{17}$

$\operatorname{Sec} A = \frac{h}{b} = \frac{17}{8}$

$\operatorname{Sec}\theta = \frac{13}{12}$

$\operatorname{Sec}\theta = \frac{h}{b} = \frac{13}{12}$

$h = 13 \qquad b = 12 \qquad P = ?$

$h^2 - 12 = P^2$

$(13)^2 - (12)^2 = P^2$

$169 - 144 = P^2$

$25 = P^2$

$5 = P$

$\sin\theta = \frac{P}{h} = \frac{5}{13}$

$\cos\theta = \frac{12}{13} \qquad \operatorname{Sec}\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12} \qquad \operatorname{Cosec}\theta = \frac{13}{5}$

$\cot\theta = \frac{12}{5}$

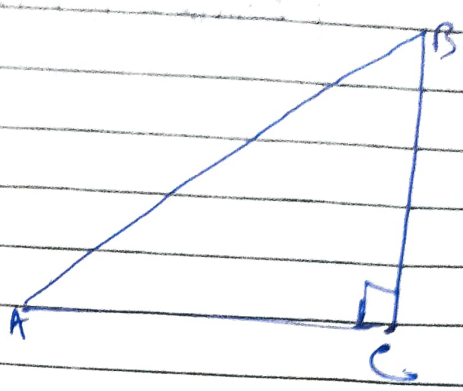
6. $\angle C = 90°$

$\cos A = \cos B$

$\frac{AC}{AB} = \frac{BC}{AB}$

$AC = BC$

$\because \angle A = \angle B$



7. $\cot\theta = \frac{7}{8}$

(a) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$(1+\cos\theta)(1-\cos\theta)$

$\cot\theta = \frac{7}{8} = \frac{AB}{BC}$

$AB = 7 \quad BC = 8$

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (7)^2 + (8)^2$

$\Rightarrow AC^2 = 49 + 64$

$\Rightarrow AC^2 = 113$

$\Rightarrow AC = \sqrt{113}$

$\sin\theta = \frac{BC}{AC} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{AB}{AC} = \frac{7}{\sqrt{113}}$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2}$$

$$= \frac{49}{113} \div \frac{49}{64}$$

$$= \frac{64}{113}$$

(ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{49}{64}$

~~(i) (ii)~~

Let ABC is a right triangle ABC

$\therefore \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let $AB = \sqrt{3}x$ and $BC = x$

$AC^2 = AB^2 + BC^2$

$= (\sqrt{3}x)^2 + (x)^2$

$\Rightarrow AC^2 = 3x^2 + x^2$

$\Rightarrow AC = \sqrt{3x^2 + x^2}$

$\Rightarrow AC = \sqrt{4x^2}$

$AC = 2h$

$\sin A = \frac{BC}{AC} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{BC}{AC} = \frac{1}{2}$

(10) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$

(11) $\cos A \cos C - \sin A \sin C =$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= 0$

12

$PR^2 = RQ^2 + PQ^2$

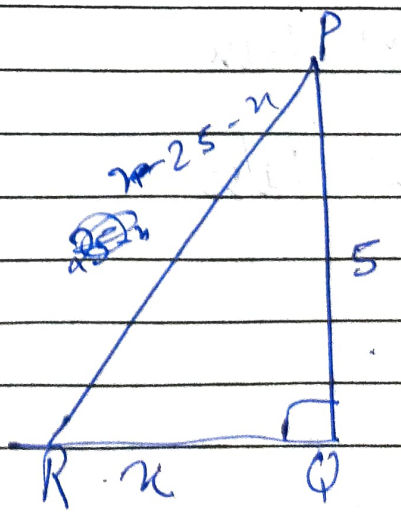
$(x-25)^2 = (x)^2 + (5)^2$

$= x^2 + 625 = x^2 + 25 + 50x$

$= +625 - 25 = 50x$

$= \frac{+600}{50} = x$

$= 12 = x$



~~$h^2 + 12^2 = 25^2$~~

$PR = 12$

$PR = 25 - 12$

$= 13$

$\sin P = \frac{P}{h} = \frac{12}{13}$

$\cos P = \frac{5}{13}$

$\tan P = \frac{12}{5}$

$\frac{1}{2}$

(i) False

(ii) True

(iii) False

(iv) False

(v) False