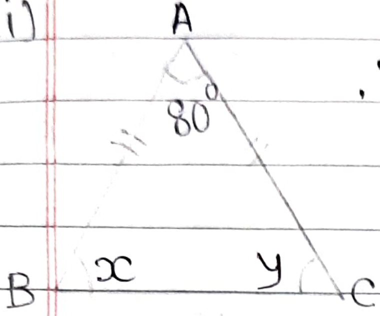
HW 4/10/21

EXERCISE 15(B)

1. Find the unknown angles in the given figures :

(i)



In triangle ABC, AB = AC

$\therefore \Delta ABC$ is an isosceles triangle.

$\angle BAC = 80^\circ$, $\angle ABC = x$, $\angle ACB = y$

$\angle ABC = \angle ACB \Rightarrow x = y$

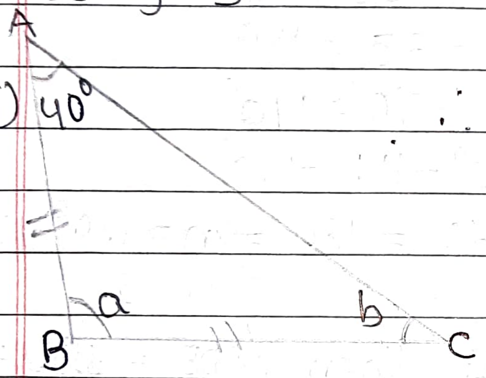
$\therefore x + y + 80^\circ = 180^\circ \Rightarrow x + x = 180^\circ - 80^\circ$

$\Rightarrow 2x = 100^\circ$

$\Rightarrow x = \frac{100^\circ}{2} = 50^\circ$

$\therefore x = y = 50^\circ$.

(ii)



In triangle ABC, AB = BC

$\therefore \Delta ABC$ is an isosceles triangle

$\angle BAC = 40^\circ$, $\angle ABC = a$, $\angle ACB = b$

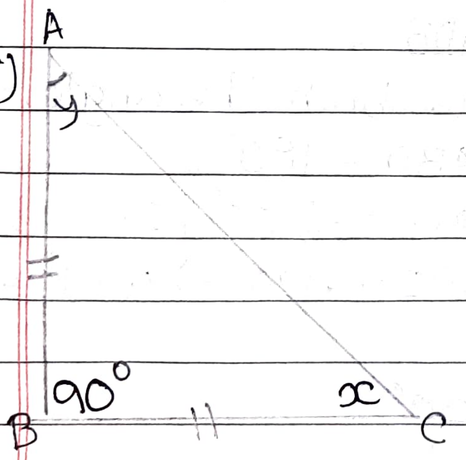
$\angle CAB = \angle ACB \Rightarrow 40^\circ = b$

$\therefore 40^\circ + b + a = 180^\circ \Rightarrow 40^\circ + 40^\circ + a = 180^\circ$

$\Rightarrow 80^\circ + a = 180^\circ \Rightarrow a = 180^\circ - 80^\circ = 100^\circ$

$\therefore b = 40^\circ$, $a = 100^\circ$

(iii)



In triangle ABC, AB = BC

$\therefore \Delta ABC$ is an isosceles triangle

$\angle ABC = 90^\circ$, $\angle BAC = y$, $\angle ACB = x$

$\therefore \angle BCA = \angle BAC \Rightarrow x = y$

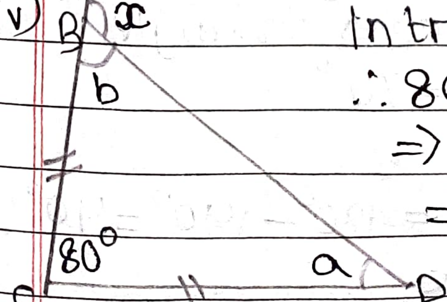
$\therefore 90^\circ + x + y = 180^\circ \Rightarrow 90^\circ + x + x = 180^\circ$

$\Rightarrow 90^\circ + 2x = 180^\circ \Rightarrow 2x = 180^\circ - 90^\circ$

$\Rightarrow 2x = 90^\circ \Rightarrow x = \frac{90^\circ}{2} \Rightarrow x = 45^\circ$

$\Rightarrow x = y = 45^\circ$

(iv)



In triangle BCD, $\angle BCD = 80^\circ$, $\angle CBD = \angle CDB \Rightarrow b = a$

$\therefore 80^\circ + b + a = 180^\circ \Rightarrow 80^\circ + b + b = 180^\circ$

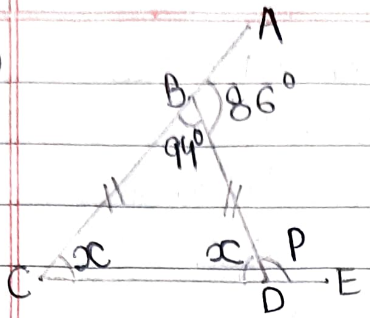
$\Rightarrow 80^\circ + 2b = 180^\circ \Rightarrow 2b = 180^\circ - 80^\circ \Rightarrow 2b = 100^\circ$

$\Rightarrow b = \frac{100^\circ}{2} \Rightarrow b = 50^\circ$, $\therefore a = 50^\circ$.

$b + x = 180^\circ \Rightarrow 50^\circ + x = 180^\circ \Rightarrow x = 180^\circ - 50^\circ$

$\Rightarrow x = 130^\circ$

(v)



In the isosceles triangle ΔBCD,

$\angle BCD = \angle BDC = x$

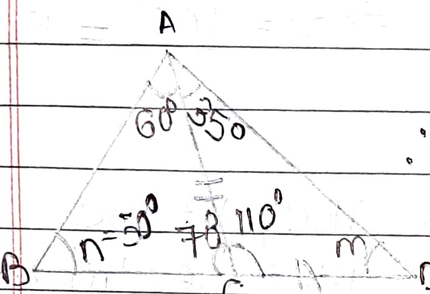
$\angle ABD = 86^\circ \Rightarrow \angle CBD = 180^\circ - 86^\circ = 94^\circ$

$\therefore 94^\circ + 2x = 180^\circ \Rightarrow 2x = 180^\circ - 94^\circ = 86^\circ$

$\Rightarrow x = \frac{86^\circ}{2} \Rightarrow x = 43^\circ$

$\therefore P = 180^\circ - x = 180^\circ - 43^\circ = 137^\circ$

(vi)



In triangle ACD, AC = CD

$\therefore \angle CAD = \angle ADC \Rightarrow 35^\circ = m$

$\therefore \angle ACD + 35^\circ + 35^\circ = 180^\circ$

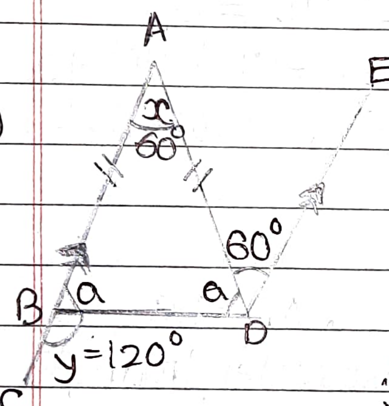
$\therefore \angle ACD = 180^\circ - 70^\circ = 110^\circ$

$\therefore \angle ACB = 180^\circ - 110^\circ = 70^\circ$

$\therefore n + 60^\circ + 70^\circ = 180^\circ \Rightarrow n = 180^\circ - 130^\circ$

$\Rightarrow n = 50^\circ$

(vii)



Since CA ∥ DE then $\angle ADE = \angle BAD = x = 60^\circ$

Since ΔABD is a isosceles treiangle.

then $\angle ABD = \angle ADB$.

Let us assume both the angles = a

$\therefore \angle BAD + \angle ADB + \angle ABD = 180^\circ$

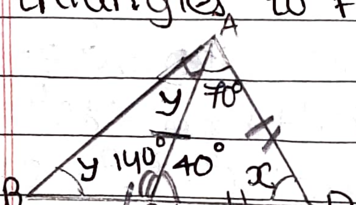
$\Rightarrow 60^\circ + a + a = 180^\circ \Rightarrow 60^\circ + 2a = 180^\circ$

$\Rightarrow 2a = 180^\circ - 60^\circ \Rightarrow 2a = 120^\circ \Rightarrow a = \frac{120^\circ}{2} = 60^\circ$

$\therefore y = 180^\circ - a \Rightarrow y = 180^\circ - 60^\circ = 120^\circ$

2. Apply the properties of isosceles and equilateral triangles to find the unknown angles in the figures

(i)



ΔCAD is a isosceles triangle

$\therefore \angle ADC = \angle x = \angle CAD = 70^\circ$

$\Rightarrow x = 70^\circ$.

$\therefore \angle ACD = 180^\circ - 2x = 180^\circ - 140^\circ = 40^\circ$

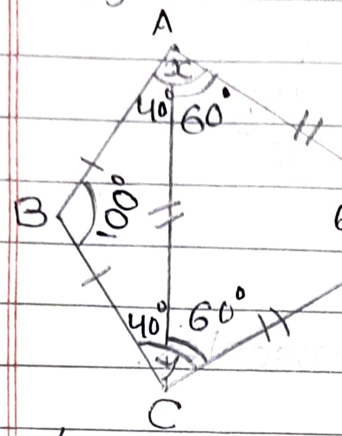
The ΔABC is an isosceles triangle,

$\therefore \angle ABC = \angle CAB = y$

$\therefore \angle ACB = 180^\circ - \angle ACD = 180^\circ - 40^\circ = 140^\circ$

In ΔABC, $140^\circ + 2y = 180^\circ \Rightarrow 2y = 180^\circ - 140^\circ \Rightarrow 2y = 40^\circ \Rightarrow y = \frac{40^\circ}{2}$
$\Rightarrow y = 20^\circ$

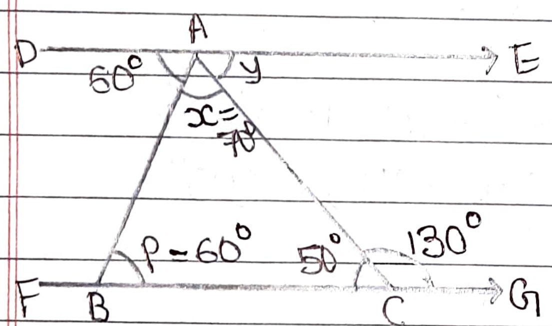
(ii)



In The ΔACD all the angles are equal i.e. 60° each. Since the triangle is a equilateral triangle.
In the ΔABC is a isosceles triangle.
$\therefore \angle ACB = \angle BAC = a$
$\therefore 100^\circ + 2a = 180^\circ \Rightarrow 2a = 180^\circ - 100^\circ$
$\Rightarrow 2a = 80^\circ \Rightarrow a = \frac{80^\circ}{2} \Rightarrow a = 40^\circ$

$\therefore \angle BCD = y = 40^\circ + 60^\circ = 100^\circ$
$\therefore \angle BAD = x = 40^\circ + 60^\circ = 100^\circ$

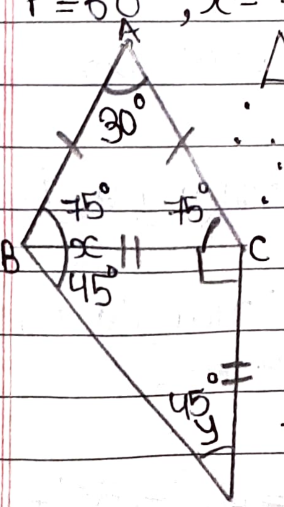
(iii)



Since the $\overrightarrow{DE}$ and $\overrightarrow{FG}$ are parallel and AB is the intersecting line of both the parallel lines. then all the interior alternate angles are equal.

$\therefore \angle ABC = p = \angle DAB = 60^\circ$
Since angle $\angle ACG = 130^\circ$ then $\angle ACB = 180^\circ - 130^\circ = 50^\circ$
$\therefore x = \angle BAC = 180^\circ - (60^\circ + 50^\circ) = 180^\circ - 110^\circ = 70^\circ$
$\therefore$ The Interior alternate angle $\angle BCA = \angle EAC = y = 50^\circ$
$\therefore p = 60^\circ, x = 70^\circ$ and $y = 50^\circ$

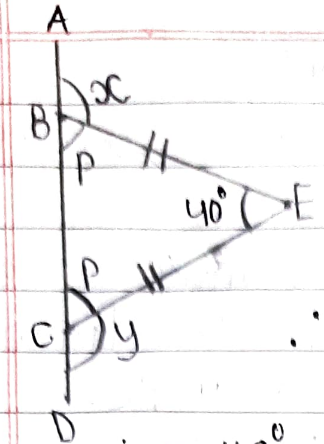
(iv)



ΔABC is an isosceles triangle.
$\therefore \angle ABC = \angle ACB = p$
$\therefore 30^\circ + 2p = 180^\circ \Rightarrow 2p = 180^\circ - 30^\circ \Rightarrow 2p = 150^\circ$
$\Rightarrow p = \frac{150^\circ}{2} = 75^\circ$

Since the ΔBCD is an Isosceles right angled triangle, then $\angle BCD = 90^\circ$ and $\angle CDB = \angle CBD = 45^\circ$
$\therefore y = 45^\circ$
$\therefore x = 75^\circ + 45^\circ = 120^\circ$

(v)



ΔBCE is an isoseceles treiangle.

$\therefore \angle EBC = \angle ECB = P$

$\therefore 2P + 40^\circ = 180^\circ, \Rightarrow 2P = 180^\circ - 40^\circ$

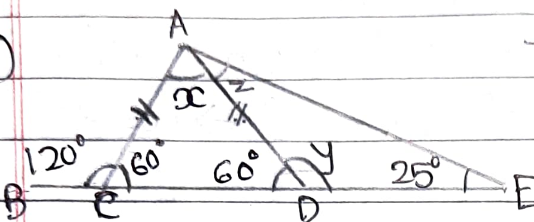
$\Rightarrow 2P = 140^\circ, \Rightarrow P = 140^\circ, \Rightarrow P = 70^\circ$

$\therefore y = 180^\circ - P, \Rightarrow 180^\circ - 70^\circ = 110^\circ$

$x = 180^\circ - P, \Rightarrow 180^\circ - 70^\circ = 110^\circ$

$\therefore x = 110^\circ, y = 110^\circ$

(vi)



The Δ ACD is isosceles.

$\therefore \angle ACD = \angle ADC$

$\angle ACB = 120^\circ,$

$\therefore \angle ACD = 180^\circ - 120^\circ = 60^\circ$

If $\angle ACD = 60^\circ$ then $\angle ADC = 60^\circ$.

$\therefore \angle CAD = 60^\circ$

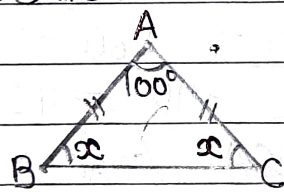
$\therefore$ If $\angle ADC = 60^\circ$ then $\angle ADE = 180^\circ - 60^\circ = 120^\circ = y$

$\therefore z = 180^\circ - (120^\circ + 25^\circ) \Rightarrow z = 180^\circ - 145^\circ = 35^\circ$

$\therefore x = 60^\circ, y = 120^\circ$ and $z = 35^\circ$

Q(3) The angle of vertex of an isosceles triangle is 100°. Find its base angles.

Ans→



The ΔABC is an isosceles triangle with a vertex angle is $= 100^\circ$. Lets assume the base angles as x.

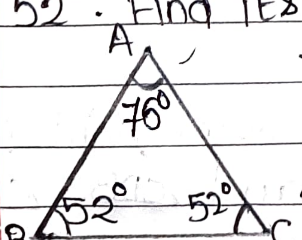
$\therefore x + x = 2x$

$\therefore 2x + 100^\circ = 180^\circ, \Rightarrow 2x = 180^\circ - 100^\circ, \Rightarrow 2x = 80^\circ, \Rightarrow x = 40^\circ$

$\therefore$ The base angles are 40° each.

Q(4) One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.

Ans→



The ΔABC is an isosceles triangle with one of the base angle $= \angle ABC = 52^\circ$.

$\therefore \angle ACB = 52^\circ,$

$\therefore$ The Angle of vertex $= \angle BAC = 180^\circ - (52^\circ + 52^\circ)$

$\Rightarrow \angle BAC = 180^\circ - 104^\circ = 76^\circ$