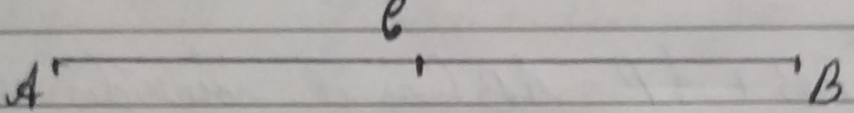
4) If a point C lies between two points A and B such that AC = BC, then prove that $AC = \frac{1}{2} AB$. Explain by drawing the figure.

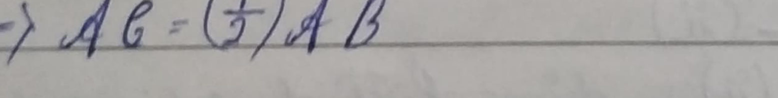
ans)



Given that, $AC = BC$.
Now, adding AC both sides.
$L.H.S + AC = R.H.S + AC$
$AC + AC = BC + AC$
$2AC = BC + AC$
We know that $BC + AC$ (as it ~~coinci~~ coincides with line segment AB)
$\therefore 2AC = AB$ (if equals are added to equals, the whole are equal.)
$\rightarrow AC = \left(\frac{1}{2}\right) AB$

5) In Question 4, ~~point~~ point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

ans)



Let, AB be the line segment
Assume that points P and Q are the two different mid points of AB.

Now,

$\because$ P and Q are the midpoints of AB.

Therefore,

$AP = PB$ and $AQ = QB$.

also,

$PB + AP = AB$ ($\because$ as it coincides with line segment AB)

Similarly, $QB + AQ = AB$.

Now,

Adding AP to the L.H.S and R.H.S of the equation $AP = PB$.

We get, $AP + AP = PB + AP$ (If equals are added to equals, the wholes are equal.)

$\Rightarrow 2AP = AB$ — (i)

Similarly,

$2AQ = AB$ — (ii)

From (i) and (ii), since R.H.S are same, we equate the L.H.S

$2AP = 2AQ$ (Things which are equal to the same thing are equal to one another.)

$\Rightarrow AP = AQ$ (Things which are double of the same things are equal to one another.)

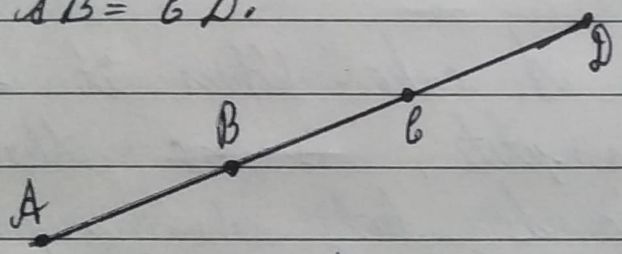
Thus, we conclude that P and Q are the same thing. This contradicts our assumption the P and Q

aus

are two different mid points of AB.
Thus, it is proved that every line segment has one and only one mid-point.
Hence Proved.

6) In Fig. 5.10, if $AC = BD$, then prove that $AB = CD$.

ans)



It is given, $AC = BD$
From the given figure, we get,
$AC = AB + BC$
$BD = BC + CD$
$\rightarrow AB + BC = BC + CD$ [$AC = BD$, given]
We know that, according to Euclid's axiom, when equals are subtracted from equals, remainders are also equal.
Subtracting BC from the L.H.S and R.H.S of the equation $AB + BC = BC + CD$, we get,
$AB + BC - BC = BC + CD - BC$
$AB = CD$
Hence Proved.

7) Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

ans) <u>Axiom 5</u>: The whole is ~~always~~ always greater than the part.

<u>For example</u>: A cake. When it ~~at~~ is whole or complete, assume that it measures 2 pounds but when a part from it is taken out ~~of~~ and measured, its weigh will be smaller than the previous measurement. So, the fifth axiom of Euclid is true for all the fifth axiom of Euclid is true for all the materials in the universe. Hence, Axiom 5, in the list of Euclid's axiom, is considered a 'universal truth'.