

HW
22/07/2021

Chapter: 8

Quadrilaterals

Exercise 8.1

- 4) Let $ABCD$ be a square and its diagonals AC and BD intersect each other at O .

To show that,

$$AC = BD$$

$$AO = OC$$

$$\text{and } \angle AOB = 90^\circ$$

Proof,

In $\triangle ABC$ and $\triangle BAD$,

$$AB = BA \text{ (Common)}$$

$$\angle ABC = \angle BAD = 90^\circ$$

$$BC = AD \text{ (Given)}$$

$$\triangle ABC \cong \triangle BAD \text{ (SAS congruency)}$$

Then,

$$AC = BD \text{ (C.P.C.T)}$$

Diagonals are equal.

Now,

In $\triangle AOB$ and $\triangle COD$,

$$\angle BAO = \angle DCO \text{ (Alternate interior angles)}$$

$$\angle AOB = \angle COD \text{ (Vertically opposite)}$$

$$AB = CD \text{ (Given)}$$

$$\triangle AOB \cong \triangle COD \text{ (AA \& congruency)}$$

Thus,

$$AO = CO \text{ (C.P.C.T)}$$

Diagonal bisect each other.

Now,

In $\triangle AOB$ and $\triangle BOD$,

$$OB = OB \text{ (Given)}$$

$$AO = CO \text{ (Diagonals are bisected)}$$

$$AB = CB \text{ (Sides of the square)}$$

$$\triangle AOB \cong \triangle COB \text{ (S.S.S congruency)}$$

Also,

$$\angle AOB = \angle COB$$

$$\angle AOB + \angle COB = 180^\circ \text{ (Linear pair)}$$

Thus,

$$\angle AOB = \angle COB = 90^\circ$$

5) Let $ABCD$ be a quadrilateral and its diagonals AC and BD bisect each other at right angle at O .

To prove that,

The Quadrilateral $ABCD$ is a square.

Proof,

In $\triangle AOB$ and $\triangle COD$,
 $AO = CO$ (Diagonals bisect each other)
 $\angle AOB = \angle COD$ (Vertically opposite)
 $OB = OD$ (Diagonals bisect each other)
 $\triangle AOB \cong \triangle COD$ (SAS congruency)

Thus,

$$AB = CD \text{ (CPCT) - (i)}$$

Also,

$$\angle OAB = \angle OCD \text{ (A.A)}$$

$$\rightarrow AB \parallel CD$$

Now,

In $\triangle AOD$ and $\triangle BOC$,
 $AO = BO$ (Diagonals bisect each other)
 $\angle AOD = \angle BOC$ (Vertically opposite)
 $OD = OC$ (Common)
 $\triangle AOD \cong \triangle BOC$ (SAS congruency)

Thus,

$$AD = BC \text{ (CPCT) - (ii)}$$

Also,

$$\rightarrow AD = BC = CD = AB \text{ - (ii)}$$

Also,

$$\angle ADC = \angle BCD \text{ (CPCT)}$$

and

$$\angle APC + \angle BCP = 180^\circ \text{ (Co-interior angles)}$$

$$\Rightarrow 2 \angle APB = 180^\circ$$

$$\rightarrow \angle APB = 90^\circ \quad \text{--- (iii)}$$

One of the interior angles is right angle,

Thus, from (i), (ii) and (iii) given quadrilateral $ABCP$ is a square.

Hence Proved.