Chapter :- 6

Lines and Angles

Exercise 6.1

1) In Fig. 6.13, lines AB and CD intersect at O. If AOC + BOE = 70° and BOD = 40°, find BOE and reflex COE.

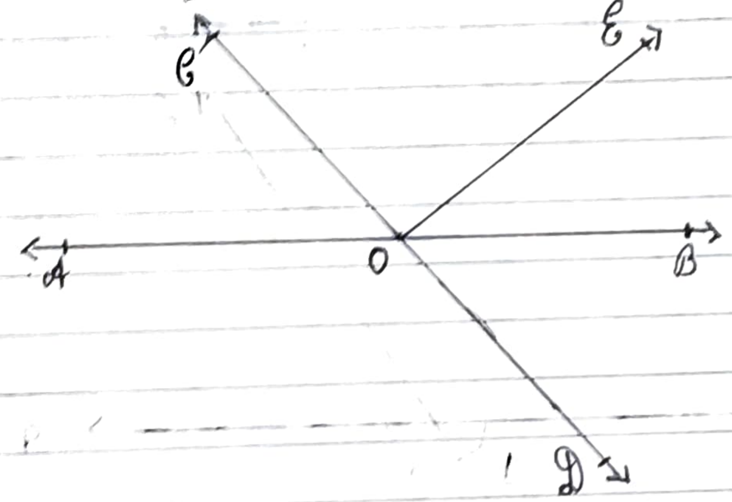


Fig. 6.13

ans) From the diagram, we have
($\angle AOC + \angle BOE + \angle COE$) and ($\angle COE + \angle BOD + \angle BOE$)
forms a straight line.
So, $\angle AOC + \angle BOE + \angle COE = \angle COE + \angle BOD + \angle BOE = 180°$
Now, by putting the values of $\angle AOC + \angle BOE = 70°$
and $\angle BOD = 40°$ we get
$\angle COE = 110°$ and $\angle BOE = 30°$
So, reflex $\angle COE = 360° - 110° = 250°$

2) In Fig. 6.14, lines XY and MN ~~intersects~~ intersect at O. If POY = 90° and a : b = 2 : 3 find c.

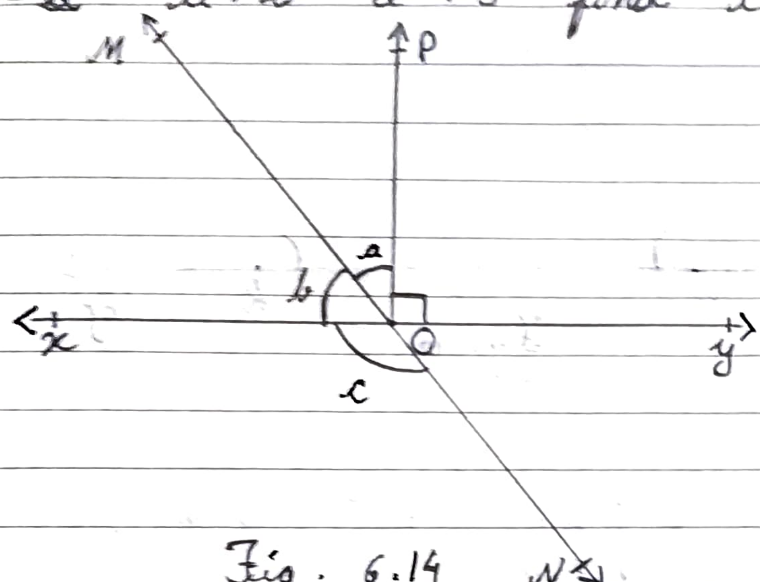


Fig. 6.14

Ans) We know that the ~~sum~~ sum of linear pair are always ~~eq~~ equal to 180°
so,

$POY + a + b = 180°$

Putting the value of $POY = 90°$ (given) we get,

$a + b = 90°$

Now, it is given that $a : b = 2 : 3$ so,
let a be $2x$ and b be $3x$

$\therefore 2x + 3x = 90$

Solving this equation we get,

$x = 18$

$\therefore 2x = 36°$ and $3x = 54°$

From the diagram, $b + c$ also forms a straight angle so,

$b + c = 180°$

$54° + c = 180°$

$c = 126°$

3) In fig 6.15, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

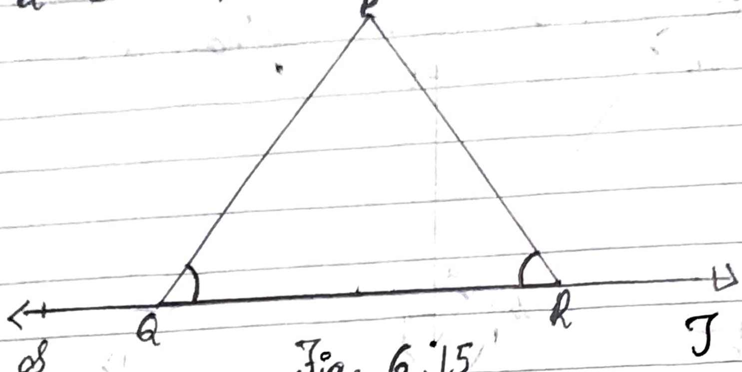


Fig. 6.15

ans) ~~Sue~~ Since ST is a straight line so,
$\angle PQS + \angle PQR = 180°$ (linear pair)
$\angle PRT + \angle PQR = 180°$ (linear pair)
Now, $\angle PQS + \angle PQR = \angle PRT + \angle PRQ = 180°$
Since $\angle PQR = \angle PRQ$ (as given in the question)
$\angle PQS = \angle PRT$. (Hence Proved)

4) In Fig. 6.16, if $x + y = w + z$, then prove that ~~A~~AOB is a line.

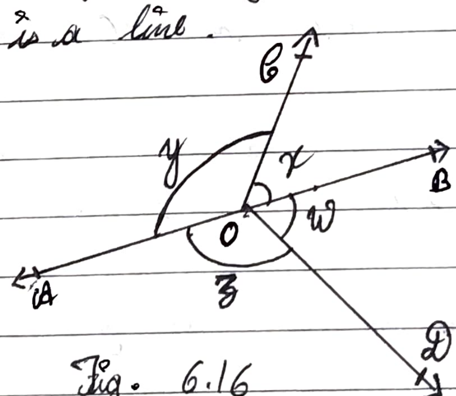


Fig. 6.16

ans) For proving AOB is a straight line, we will have to prove $x + y$ is a linear pair.
i.e. $x + y = 180°$
We know that the angles around a point are $360°$ so,
$x + y + z + w = 360°$
given,
$x + y = w + z$ So, $(x + y) + (x + y) = 360°$
$2(x + y) = 360°$ ∴ It is proved that $(x + y) = 180°$

5) In Fig. 6.17, ~~PQ~~ POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $ROS = \frac{1}{2}(QOS - POS)$.

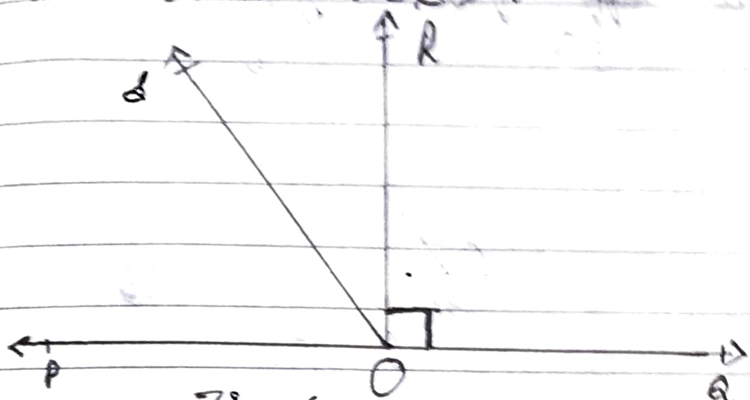


Fig. 6.17

Ans) In this question, it is given that $(OR \perp PQ)$ and $POQ = 180^\circ$

So,

$$POS + ROS + ROQ = 180^\circ$$

Now,

$$POS + ROS = 180^\circ - 90^\circ \text{ (Since } POR = ROQ = 90^\circ)$$

$\therefore POS + ROS = 90^\circ$

Now,

$$QOS = ROQ + ROS$$

It is given that $ROQ = 90^\circ$

$\therefore QOS = 90^\circ + ROS$

Or,

$$QOS - ROS = 90^\circ$$

As $POS + ROS = 90^\circ$ and $QOS - ROS = 90^\circ$, we get

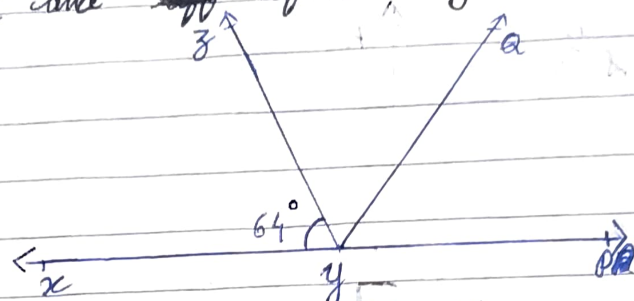
$POS + ROS = QOS - ROS$

$2\,ROS + POS = QOS$

Or, $ROS = \frac{1}{2}(QOS - POS)$ $\therefore$ proved.

6) It is given that $XYZ = 64°$ and XY is produced to point P. Draw a figure from the given ~~refer~~ information. If a ray YQ bisects ZYP, find XYQ and ~~reff~~ reflex QYP.

ans)



Here, XP is a straight line
So,

$$XYZ + ZYP = 180°$$

Putting the value of $XYZ = 64°$ we get,

$64° + ZYP = 180°$

$\therefore ZYP = 116°$

From the diagram, we also know that ~~$ZYP = ZYQ + QYP$~~

~~Now~~ $ZYP = ZYQ + QYP$

Now,

as YQ bisects ZYP,

$ZYQ = QYP$

Or, $ZYP = 2ZYQ$

$\therefore ZYQ = QYZ + ZYQ$

By putting the value of $XYZ = 64°$ and $ZYQ = 58°$ we get,

$$XYQ = 64° + 58°$$

Or, $XYQ = 112°$

Now, Reflex $QYP = 180° + XYQ$

We computed that the value of $XYQ = 112°$

So,

$QYP = 180° + 112°$

$\therefore QYP = 302°$

Exercise 6.2

1) In Fig. 6.28, find the values of x and y and then show that $AB \parallel CD$.

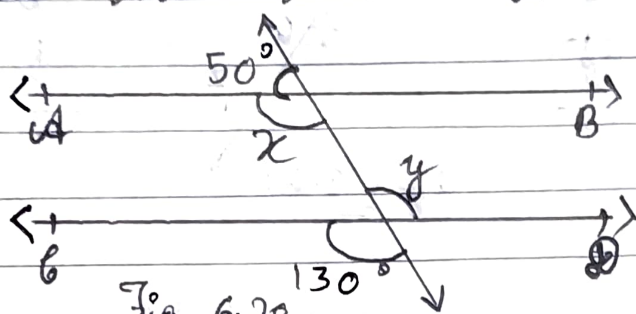


Fig. 6.28

ans) We know that a linear pair is equal to 180°

So, $x + 50° = 180°$

$x = 130°$.

We ~~know the~~ also know that vertically ~~B~~ Opposite angles are equal.

So,

$y = 130°$

In two parallel ~~angles~~ lines, the alternate interior angles are equal. In this

$x = y = 130°$

∴ This proves that alternate interior ~~angel~~ angles are equal and so, it is proved that $AB \parallel CD$.

2) In Fig. 6.29, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.

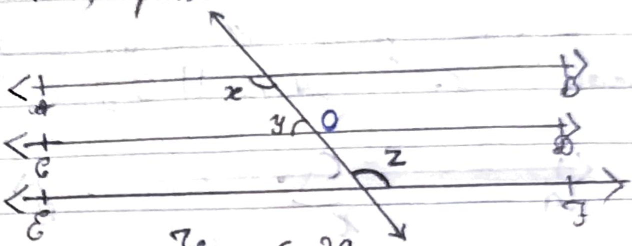


Fig. 6.29

ans) It is known that ~~AB CD~~ $AB \parallel CD$ ~~ab~~ and $CD \parallel$ [illegible] EF

As the angles on the same side of a transversal line sums up to 180°.

$x + y = 180^\circ$ (i)

Also,

$O = z$ (since they are corresponding angles)

and, $y + O = 180^\circ$ (since they are linear pair)

So,

$y + z = 180^\circ$

Now, let $y = 3w$ and hence, $z = 7w$ (As $y : z = 3 : 7$)

$\therefore 3w + 7w = 180^\circ$

Or, $10w = 180^\circ$

So,

$w = 18^\circ$

Now, $y = 3 \times 18^\circ = 54^\circ$

and, $z = 7 \times 18^\circ = 126^\circ$

Now, angle x can be calculated from equation (i)

$x + y = 180^\circ$

Or, $x + 54^\circ = 180^\circ$

$\therefore x = 126^\circ$

3) In fig. 6.30, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find AGE, GEF, FGE.

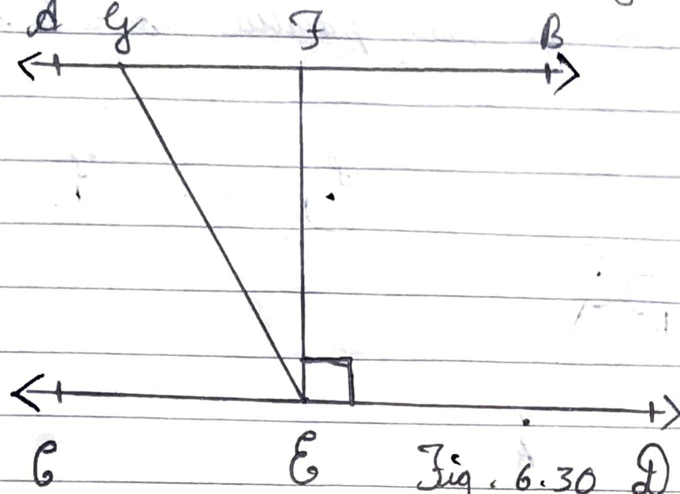


Fig. 6.30

Ans) Since $AB \parallel CD$, GE is a transversal.
It is given the $GED = 126^\circ$
So,

$GED = AGE = 126^\circ$ (As they are alternate interior angles)

Also,

$GED = GEF + FED$

As,

$EF \perp CD$, $FED = 90^\circ$

$\therefore GED = GEF + 90^\circ$

Or,

$GEF = 126^\circ - 90^\circ = 36^\circ$

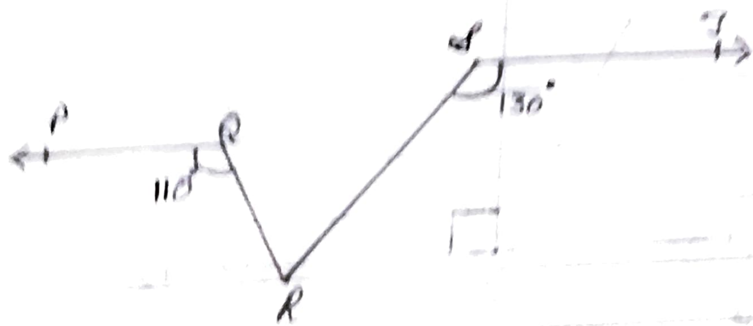
Again,

$FGE + GED = 180^\circ$ (Transversal)

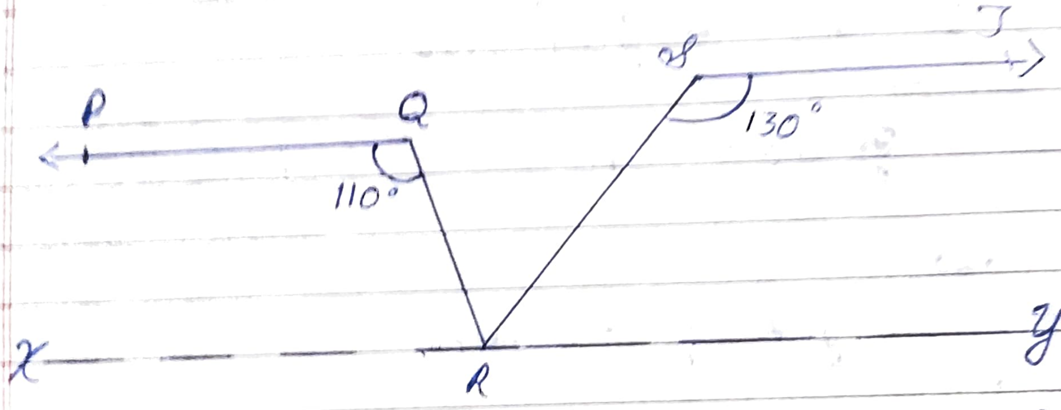
Putting the value of $GED = 126^\circ$ we get,
$FGE = 54^\circ$
So,

$AGE = 126^\circ$
$GEF = 36^\circ$
$FGE = 54^\circ$

4) In Fig. 6.31, if $PQ \parallel ST$, $PQR = 110^\circ$ and $RST = 130^\circ$, find QRS.
[Hint: Draw a line parallel to ST through point R.]



Ans) First construct a line xy parallel to PQ.



We know that the angles on the same side of the transversal is equal to 180°

So, $PQR + QRX = 180^\circ$

Or, $QRX = \cancel{70^\circ}\ 180^\circ - 110^\circ$

$= 70^\circ$

Similarly,

$RST + SRY = 180^\circ$

Or, $SRY = 180^\circ - 130^\circ$

$= 50^\circ$

Now, for the linear pairs on the line xy: $QRX + QRS + SRY = 180^\circ$

Or, $QRS = 180^\circ - 70^\circ - 50^\circ$

$= 60^\circ$

5) In Fig 6.32, if $AB \parallel CD$, $APQ = 50^\circ$ and $PRD = 127^\circ$, find x and y.

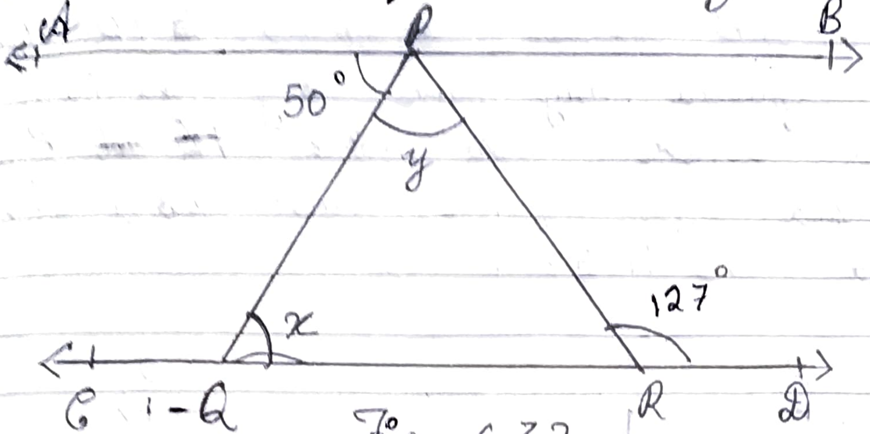


Fig. 6.32

ans) From the diagram,

$APQ = PQR$ (Alternate interior angles)

Now, putting the value of $APQ = 50^\circ$ & $PQR = x$ we get

$x = 50^\circ$

Also,

$APR = PRD$ (Alternate interior angles

Or, $APR = 127^\circ$

We know that

$APR = APQ + QPR$

Now, putting values of $QPR = y$ and $APR = 127^\circ$

We get,

$127^\circ = 50^\circ + y$

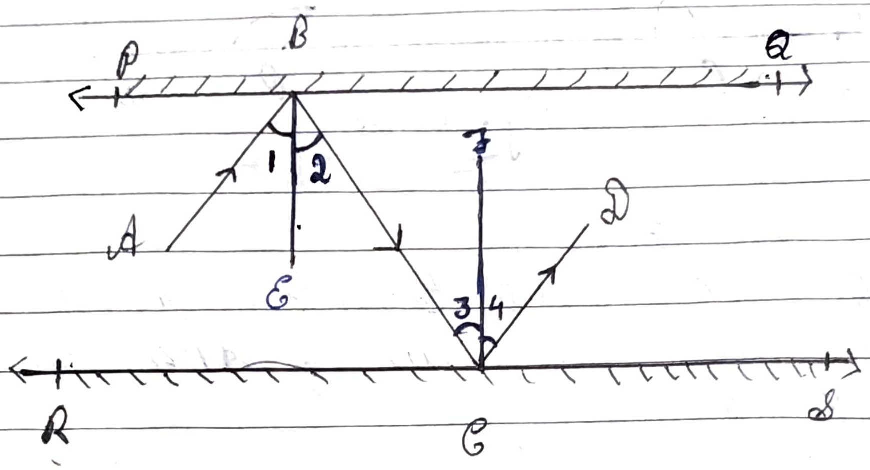
Or, $y = 77^\circ$

Thus, the value of x and y are calculated as:

$x = 50^\circ$

$y = 77^\circ$

6) In Fig. 6.33, PQ and RS are two mirrors parallel to each other. An ~~ancient~~ incident ray AB strikes the mirror ~~B~~ PQ at B, the reflected ray moves along the Path BC and strikes the mirror ~~PQ at~~ RS at C and again reflects back along CD. Prove that AB ∥ CD.



We know that,

Angle of ~~incident~~ incidence = Angle of reflection (by the law of Reflection)

So,

1 = 2 and

3 = 4

We also know that alternate interior angles are equal. Here, BE ⊥ CF and the transversal line BC cuts them at B and C

So, 2 = 3 (Alternate interior angles)

Now,

1 + 2 = 3 + 4

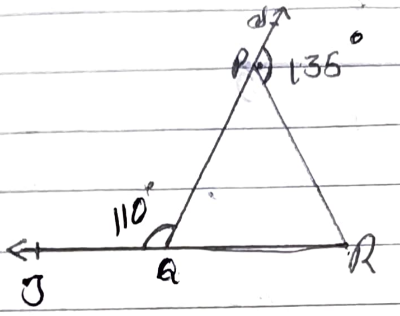
Or, ABC = DCB

So,

AB ∥ CD because alternate interior angles are equal.

Exercise 6.3

1) In Fig. 6.39 sides QP and RQ ~~are~~ of ΔPQR ~~PR~~ are produced to point S and T respectively. If $SPR = 135°$ and $PQT = 110°$, find PQR.



Ans. It is given the TQR in a straight line and so, the linear pairs (i.e. TQP and PQR) will add up to 180°.

So,

~~Now~~ $TQP + PQR = 180°$

Now, Putting the value of $TQP = 110°$ we get,

$PQR = 70°$

Consider that ΔPQR,

Here, the side QP is extended to S

and so, SPR form the exterior angles.

Thus,

SPR ($SPR = 135°$) is equal to the sum of interior opposite angles. (Triangle property)

Or, $PQR + PRQ = 135°$

Now, Putting the value of $PQR = 70°$ we get,

$PRQ = 135° - 70°$

Hence, ~~PQ~~ $PRQ = 65°$

2) In Fig 6.40, $X = 62°$, $XYZ = 54°$. If YO and ZO are the bisectors of XYZ and XZY respectively of ΔXYZ find OZY and YOZ.

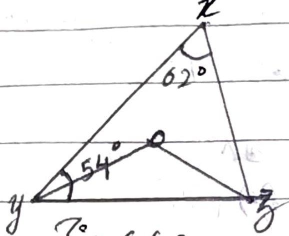


Fig 6.40

ans) We know that the sum of the interior angles of triangle.

So, $X + XYZ + XZY = 180°$

Putting the values as given in the question we get,

$62 + 54° + XZY = 180°$

$\Rightarrow XZY = 64°$

Now ~~putting~~ we ~~know~~ know that ZO is the bisector ~~so~~, $OZY = 1/2\ XYZ$

$OZY = 32°$

Similarly, YO is a bisector and ~~so~~,

$OYZ = 1/2\ XYZ$

$\Rightarrow OYZ = 27°$

Now, as the sum of the interior angles of the triangle,

$OZY + OYZ + O = 180°$

Putting their respective values, we get,

$O = 180° - 32° - 27°$

$\Rightarrow O = 121°$

3) In Fig. 6.41, if AB ∥ DE, BAC = 35° and CDE = 53°, find DCE.

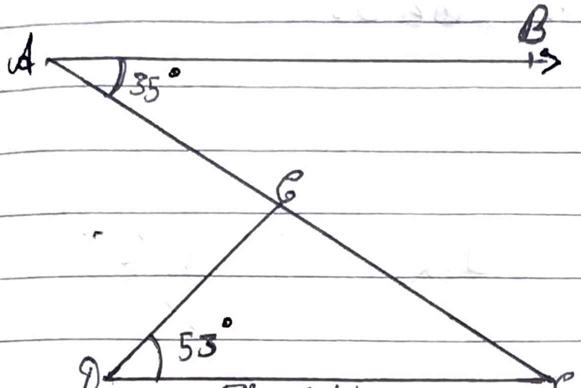


Fig. 6.41

ans) We ~~know~~ know that AE is a transversal since AB ∥ DE
Here BAC and AED are alternate interior angles.
Hence, BAC = AED
It is given that BAC = 35°
AED = 35°
Now consider the triangle CDE. We know that the sum of the ~~inner~~ interior angles of a triangle is 180°.
∴ DCE + CED + CDE = 180°
Putting the values, we get
DCE + 35° + 53° = 180°
→ DCE = 92°

4) In Fig. 6.42, if lines PQ and RS intersects at point T, such that PRT = 40°, RPT = 95° and TSQ = 75°, find SQT.

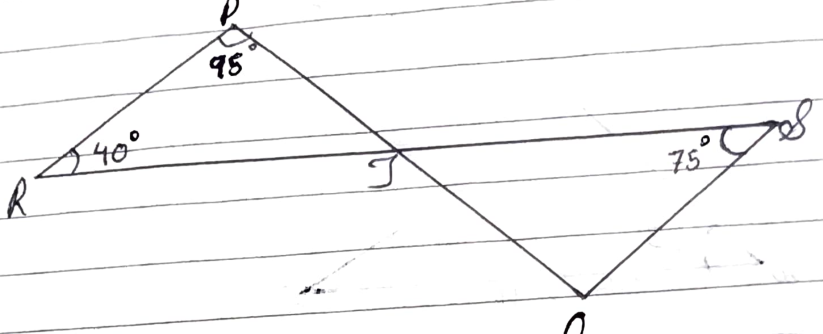


Fig. 6.42

ans) Consider ~~the~~ triangle PRT.
PRT + RPT + PTR = 180°
~~Now~~ PTR will be equal to STQ as they are vertically opposite angles.
So, PTR = STQ = 45°
Again, in triangle STQ,
TSQ + PTR + SQT = 180°
Solving this we get,
SQT = 60°

5) In Fig. 6.43, if PQ ⊥ PS, PQ ∥ SR, ∠SQR = 28° and ∠QRT = ~~65°~~ and ∠QRT = 65°, then find the value of x and y.

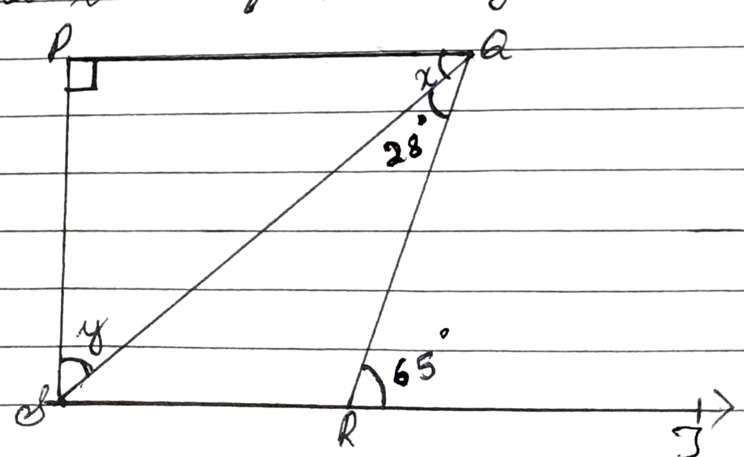


Fig. 6.43

ans) $x + SQR = QRT$ (As they are alternate interior angles since QR is transversal)

So, $x + 28^\circ = 65^\circ$

$\Rightarrow x = 37^\circ$

$QRS = x = 37^\circ$ (As they are alternate interior angles ~~and~~ since QS is transversal)

Also, Now,

$QRS + QRT = 180^\circ$ (As they are linear pair)

$\Rightarrow QRS + 65^\circ = 180^\circ$

$\Rightarrow QRS = 115^\circ$

~~In $\triangle SPQ$~~,

~~$\angle SPQ + x + y =$~~

Now, we know that the sum of the angles in a quadrilateral is 360°.

So,

$\cancel{PQR}\ P + Q + R + S = 360^\circ$

$\Rightarrow 90^\circ + 65^\circ + 115^\circ + S = 360^\circ$

$\Rightarrow S = 360^\circ - 280^\circ$

$\Rightarrow S = 80^\circ$

~~$y = S$ (as PQ $\perp$ PS and~~

Now, we know that sum of all interior angles of a triangle is 180°.

So,

$P + Q + S = 180^\circ$

$\Rightarrow 90^\circ + 37^\circ + S = 180^\circ$

$\Rightarrow S = 180^\circ - 127^\circ$

$\Rightarrow S = 53^\circ$

Hence $y = 53^\circ$